PHYSICS NOTES
FORM 3 TUTORIAL

TOPIC 1.: LINEAR MOTION

1.1: Introduction

The study of motion is divided into two areas namely kinematics and dynamics. Kinematics deals with the motion aspect only while dynamics deals with the motion and the forces associated with it.

There are three common types of motion:

- Linear or translational motion.
- Circular or rotational motion.
- Oscillatory or vibrational motion.

In this topic, we concentrate on linear motion.

Note that all motion is relative i.e the state of a body; at rest or in motion, is **ONLY** true with respect to the observer's position.

1.2: Terms associated with linear motion

- **Distance-** is the length of the path covered by a body. It only gives the magnitude but no direction i.e it is a scalar quantity.
- **Displacement-** is the distance through which a body travels in a specified direction. It is a vector quantity.

Both distance and displacement are measured in metres.

- **Speed-** is the distance covered per unit time.

Speed= distance/time.

- **Velocity-** is the rate of change of displacement.

Velocity= displacement/time.

It is a vector quantity.

When the rate of change of displacement is non-uniform, we talk about average velocity;

Average velocity= total displacement/total time.

Both speed and velocity are expressed in metre per second (m/s).

- **Acceleration-** is the rate of change of velocity.

Thus, Acceleration= change in velocity/time interval = (final velocity v - initial veolicity u)/time.

Acceleration is measured in metre per square second (m/s^2).

If the velocity of a body decreases with time, its acceleration becomes negative. A negative acceleration is referred to as **deceleration or retardation**.

Example 1.1

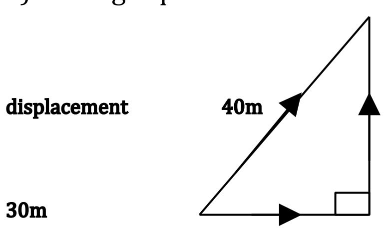
1. A body covers a distance of 2m in 4seconds, rests for 2seconds and finally covers a distance of 90m in 6seconds. Calculate its average speed.

Average speed= total distance/time= (2m+90m)/(4s+2s+6s)

= 20m/20s =5m/s.

5. A body moves 30m due east in 2seconds, then 40m due north in 4seconds. Determine its:

a) Average speed.



Average speed= total distance/time= (30m+40m)/(2s+4s)
=70m/6s =11.67m/s.

a) Average velocity.
b) Average velocity= total displacement/time =50m/6s

=6.33m/s.
3. A body is made to change its velocity from 20m/s to 36m/s in 0.1s. What is the acceleration produced?
a= (v-u)/t =(36m/s – 20m/s)/0.1s
=30m/s$^{5.}$
1. A particle moving with a velocity of 200m/s is brought to rest in 0.02s. What is the acceleration of the particle?
a= (v-u)/t =(0m/s-200m/s)/0.02
= -200/0.02 = -2,000m/s$^{5.}$

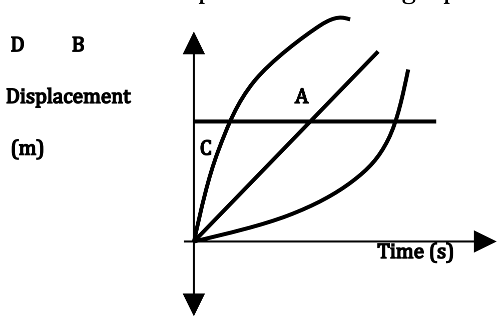
1.3: Motion graphs.

There are two categories; displacement-time graphs and velocity time graphs.

1.3.1: Displacement-time graphs

The slope of a displacement-time graph gives the velocity of the body.

The various displacement-time graphs are as illustrated below:



Graph A: the body is at rest i.e there is no change in displacement as time changes. The slope of the graph and hence the velocity is zero.

Graph B: the body moves with a uniform or constant velocity.

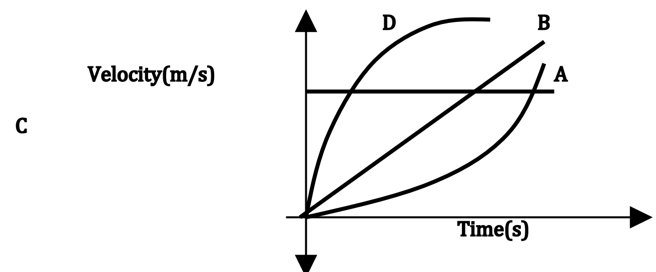
Graph C: the graph becomes steeper with time. The steeper the slope, the higher the velocity. Thus velocity of the body increases with time. The body is therefore accelerating.

Graph D: the graph becomes less and less steep with time i.e the body has a higher velocity at the beginning and decreases with time. Therefore, the body is said to be decelerating.

1.3.2: Velocity-time graphs

The slope of a velocity-time graph gives the acceleration of the body. Note that the area under a velocity-time graph gives the distance covered by the body.

The diagram below shows the possible velocity-time graphs:



Graph A: the velocity remains constant/uniform as time increases. The slope of the graph and hence the acceleration of the body is zero.

Graph B: the velocity changes uniformly with time. The body moves with a uniform/constant acceleration.

Graph C: the acceleration is lower where the graph is gentle and higher where the graph is steeper. Hence the acceleration of the body increases with time.

Graph D: in this case, the graph is steeper at the beginning and becomes gentle with time. Hence the acceleration of the body decreases with time.

1.4: Determination of velocity and acceleration

Two methods are applicable here:

Method 1: Using appropriate instruments e.g a tape measure and a stop watch to measure the displacement of a body and the duration then applying the formula;

Velocity= total displacement/time taken.

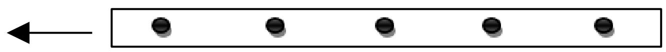
Method 2: Using a ticker-timer. It is used to measure velocity of a body specifically over short distances. It consists of an electronic vibrator which makes dots on a moving paper tape attached to the object whose velocity is being measured. The dots are made at a certain set frequency. For instance, a ticker-timer whose frequency is 50Hz makes dots at intervals of 0.02s. The time interval between successive dots is referred to as a **tick**.

The spacing between the dots depends on the manner in which the body is moving i.e moving at constant velocity or with increasing velocity or decreasing velocity. Generally, the dots are close together when the velocity is low and wide apart when the velocity is high. There are three possible patterns that can be obtained by a ticker-timer as illustrated below:

a) Moving at constant velocity.

The dots are equally or evenly spaced.

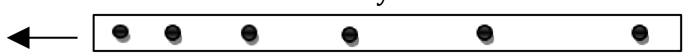
Direction of motion of the body



b) Moving with increasing velocity (accelerating).

The spacing between the dots is initially small but increases away.

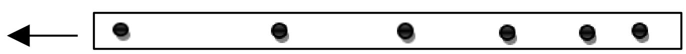
Direction of motion of the body



c) Moving with decreasing velocity (decelerating).

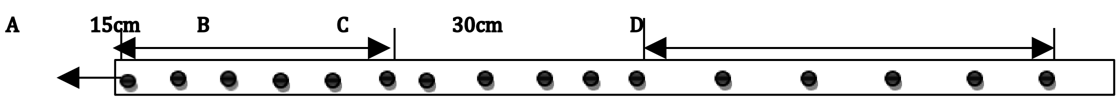
The spacing between the dots is initially large but decreases away.

Direction of motion of the body



Example 1.2

1. A paper tape was attached to a moving trolley and allowed to run through a ticker-timer. The figure below shows a section of the tape.



If the frequency of the ticker-timer is 20Hz, determine:

a) The velocity between AB and CD.

1tick= 1/20 =0.01s

V_{AB}= 15cm/(5ticks*0.01s) =15cm/0.05s

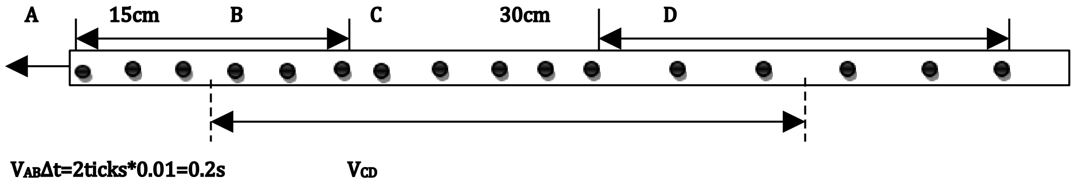
=300cm/s

V_{CD}=30cm/(5ticks*0.01s) =30cm/0.05s

=600cm/s

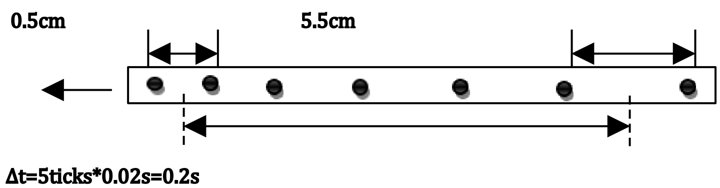
b) The acceleration of the trolley.

Note that the velocities calculated in (a) above are average velocities and as such are taken to be the velocities at the midpoints of AB and CD respectively. Hence, the time taken for the change in velocity is the time between the midpoints of AB and CD.



Therefore, acceleration=(V_{CD}- V_{AB})/Δt= (600-300)cms^{-1}/0.2s =3000$cms^{-5.}$

5. The figure below represents part of a tape pulled through a ticker-timer by a trolley moving down an inclined plane. If the frequency of the ticker-timer is 50Hz, calculate the acceleration of the trolley.



Note that 1tick=1/50 =0.02s.

Initial velocity u =0.5cm/0.02s= 25cms^{-1}

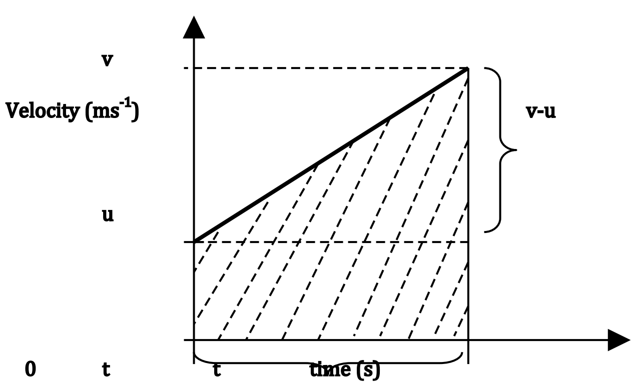
Final velocity v =5.5cm/0.02s= 125cms^{-1}

Hence, acceleration= (v-u)/Δt=(125-25)cms^{-1}/0.2s

=200cms^{-2}

1.5: Equations of linear motion

There are three equations governing linear motion. Consider a body moving in a straight line from an initial velocity u to a final velocity v(u, v≠0) within a time t as represented on the graph below:



The slope of the graph represents the acceleration of the body;

Acceleration, a=(v-u)/t.

Therefore, v=u+at………………………………. i.

This is the first equation of linear motion.

The area under the graph (area of a trapezium) gives the displacement of the body.

Hence, displacement s= ½(sum of // sides)*perpendicular height between them.

s= ½(u+v)t.

But v=u+at,

Therefore, s=½{u+(u+at)}t

s=½(2u+at)t

Hence, $s=ut+½at^2$……………………………… ii.

This is the second equation of linear motion.

Also, rearranging equation i, we have t=(v-u)/a. substituting this in equation ii, we obtain;

$s=ut+½at^2=u\{(v-u)/a\}+½a\{(v-u)/a\}^{5.}$

$s=u(v-u)/a + a(v-u)^2/2a^2= u(v-u)/a + (v-u)^2/2a$

$s= \{2u(v-u) + (v-u)^2\}/2a = \{2uv-2u^2+v^2+u^2-2uv\}/2a$

$s= \{v^2-u^2\}/2a$

$2as= v^2-u^2$

Hence, $v^2=u^2+2as$ ……………………………… iii.

This is the third equation of linear motion.

The three equations hold for any body moving with uniform acceleration.

Note that for a body which is retarding, the acceleration a is given a negative sign.

Example 1.3

1. A particle travelling in a straight line at 2m/s is uniformly accelerated at $5m/s^2$ for 8seconds. Calculate the displacement of the particle.

$s=ut+½at^2=(2*8)+(½*5*8^2)$

=176m.

5. An object accelerates uniformly at $3ms^{-5.}$ It attains a velocity of 4m/s in 5seconds.

a) What was its initial velocity?

v=u+at

u= 4-(3*5)= 4-15 =4m/s.

b) How far does it travel during this period?

$s=ut+½at^2= (4*5)+(½*3*5^2)= 53.5m$

3. A car travelling at 20m/s decelerates uniformly at $4m/s^{5.}$ In what time will it come to rest?

v=u-at, (a is negative since the body is decelerating).

0=20-4t

t=20/4 =5seconds.

1.6: Motions under the influence of gravity

These include free fall, vertical projection and horizontal projection. The three equations of linear motion hold for motions under the influence of gravity.

1.5.1: Free fall

A body falling freely in a vacuum starts from an initial velocity zero and accelerates at approximately $9.8ms^{-2}$ towards the centre of the earth. This is called the acceleration due to gravity **g**. In this case, the air resistance is assumed to be negligible. Note that in a vacuum, a feather and a stone released from the same height will take the same amount of time to reach the surface of the earth.

Therefore, in the three equations of linear motion u=0m/s, s=h and a=g. thus the three equations become:

- ✓ v=gt, (from v=u+at)
- ✓ $h=½gt^2$, (from $s=ut+½gt^2$)
- ✓ $v^2=2gh$, (from $v^2=u^2+2as$)

From the above equations:

- $v= (2gh)^{½}$, where v is the velocity of the body just before it hits the ground.
- $h=½gt^2=v^2/2g$, where h is the height through which the body falls.
- $t=v/g=(2h/g)^{½}$, where t is the time of flight.

Example 1.4

1. A hammer falls from the top of a building 5m high.

a) How long does it take to reach the ground? Take $g=2ms^{-5.}$

$h=½gt^2$

$5=½*2t^2$

$t=1^{½}=1s$

b) With what velocity does it strike the ground?

$v= (2gh)^{½}= (2*2*5)^{½}=2m/s.$

1.5.2: Vertical projection

When a body is projected vertically upwards, it decelerates uniformly due to gravity until its velocity reduces to zero at maximum height. After attaining the maximum height, the body then falls back with an increasing velocity. The body must be given an initial velocity and attains a final velocity of zero at its maximum height. Note that the sign of 'g' is negative for a vertical projection. This is because the body moves against gravity.

Hence the three equations of linear motion become:

- ✓ $v=u-gt$, (from $v=u+at$)
- ✓ $h=ut-½gt^2$, (from $s=ut+½gt^2$)
- ✓ $v^2=u^2-2gh$, (from $v^2=u^2-2as$)

But at maximum height h_{max}, v=0. Thus, the three equations reduce to:

i. $gt=u$,

ii. $h=ut-½gt^2$

iii. $u^2=2gh$.

From equation (i), the time taken to attain the maximum height is given by;

$t=u/g$.

Similarly, the initial velocity u and the maximum height attained by the body h_{max} can be expressed as:

$u=gt=(2gh_{max})^{½}$

And $h_{max}=ut-½gt^2=u^2/2g$.

When the body finally falls back to its point of projection, the displacement of the body will be zero. Substituting this in equation (ii), we obtain;

$0=ut-½gt^2$

Therefore, $0=t(2u-gt)$

And t=0 or $t=2u/g$, where t=0 is the time at the start of the projection and,

t is this is the total time of flight i.e for both upward projection and fall back. Note that the total time of flight is twice the time taken to attain maximum height.

also, the velocity of the body just before hitting its point of projection as it falls back is the same in magnitude but in opposite direction to its initial velocity; $v=-u$.

Example 1.5

1. A bullet is shot vertically upwards and rises to a maximum height of 200m. Calculate:

a) the initial velocity of the bullet,

$u=(2gh_{max})^{½}= (2*2*200)^{½}=$

b) the total time of flight.

t=2u/g=2*

5. An object is released to fall vertically from a height of 20m. At the same time, another object is projected vertically upwards with a velocity of 40m/s.

a) Calculate the time taken before the two objects meet.

Let the time taken to meet be t. then, after a time t the distance covered by the object moving downwards will be; $s_d=\frac{1}{2}gt^2$, (since u=0).

$=\frac{1}{2}*2t^2=5t^2$

The distance covered by the object projected upwards after a time t will be;

$s_u=ut-\frac{1}{2}gt^2=40t-5t^2$

But $s_d+s_u=20m$

Therefore, $5t^2+40t-5t^2=20$

t=20/40=5.5s

b) At what height above the point of projection do they meet?

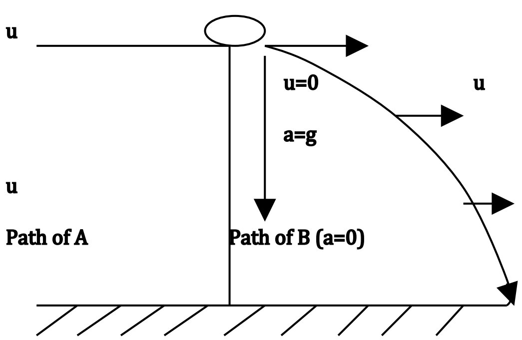
$s_u=ut-\frac{1}{2}gt^2=(40*5.5)-(\frac{1}{2}*2*5.5^2)$

=66.75m

1.5.3: Horizontal projection

If two objects A and B at a point some height above the ground are such that A is allowed to fall freely (vertically downwards) while is B given a horizontal projection with an initial velocity u, then both objects take the same duration to reach the ground. This is because both are acted on by the same gravitational force. The object on the horizontal projection moves with a constant velocity u. hence, the horizontal acceleration of the object is zero. For the object falling freely, the acceleration is equivalent to 'g' and the initial velocity u is zero. However, the object under horizontal projection will strike the ground some distance away from the point the other object strikes the ground. This horizontal distance covered by the object is referred to as the '**range R**'.

Note that both A and B will strike the ground with the same velocity.



Since a=0 for the horizontal projection, s=R=ut.

Also, the time taken to reach the ground in both cases is expressed as;

$t=u/g$.

Example 1.6

1. A stone is thrown at a velocity of 30m/s to the horizontal by a girl at the top of a tree whose height is 30m. Calculate:

a) the time taken for the stone to strike the ground.

Since both free fall and horizontal projection take the same duration;

$h=\frac{1}{2}gt^2$

$30=\frac{1}{2}*2*t^2$

$t=6^{\frac{1}{2}}=$

b) the velocity at which the stone strikes the ground.

$u=0$ (for free fall).

Therefore, $v=(2gh)^{\frac{1}{2}}=(2*2*30)^{\frac{1}{2}}$

$=$

5. A jet fighter on practice moving at a velocity of 20m/s released a bomb above the ground which hits the ground after 3s. Calculate:

a) the distance from the ground to the jet,

$h=\frac{1}{2}gt^2=\frac{1}{2}*2*3^2$

$=45m$

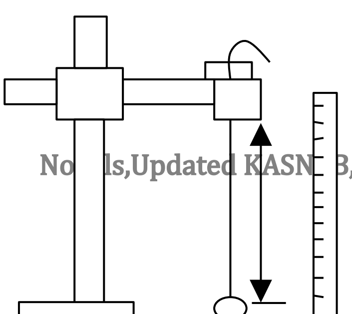
b) the horizontal distance from the target when the bomb is released.

$R=ut=20*3$

$=300m$.

1.7: Experimental determination of acceleration due to gravity.

This can be done as follows:



L Metre rule

- Set the apparatus as shown in the diagram above. Set the length of the string at 30cm. note that the length l is measured from the centre of the bob.

- Displace the bob sideways through a small angle of about 2^0 and release it so as to oscillate.

- With the help of a stop watch, measure and record the time for ten oscillations (allow some little oscillations after release before timing). Repeat this step twice or thrice and determine the average time.

Hence calculate the period T(time for one oscillation).

- Repeat the above steps for l=40cm, 50cm, 60cm, 70cm and 80cm. complete the table below:

Length,l (cm)	Time for 2oscillations, t(s)				Period, T(s)	$T^2(s^2)$
	t_1	t_2	t_3	$t=(t_1+t_2+t_3)/3$		

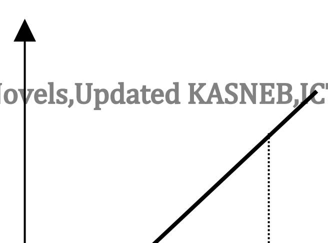
- plot a graph of T^2 against length l in metres.

Observations and conclusion

The frequency of oscillation increases with decrease in length of the string. A graph of T^2 against length l is a straight line through the origin.

Generally, a graph of T^2 against length for a simple pendulum satisfies the equation $T^2=4\pi^2 l/g$.

Hence, the slope of the graph above is equals to $4\pi^2/g$.



$T^2(s^2)$

Slope=$4\pi^2/g$

0 **Length(m)**

TOPIC 2.: NEWTON'S LAWS OF MOTION

2.1: Introduction

The laws governing the motion of a body are grouped into three. They are based on the effects of force on a body. Some of the effects of force on a body include:

- Force can make a stationary body to start moving.
- Can make a moving to stop.
- Can deform a body i.e. change its shape.
- Can change the direction of a moving body.
- Can change the speed of a moving body.

3.2: Newton's first law of motion

The law states: a body remains in its state of rest or uniform motion in a straight line unless acted upon by an external force. This explains the following common observations:

- Passengers in a bus are pushed forward when brakes are applied suddenly or backwards when a bus at rest takes off suddenly. Hence the fitting of seatbelts in vehicles.
- A coin placed on a cardboard on top of a glass tumbler drops into the tumbler when the cardboard is pulled sideways.
- Athletes run past the finish line of a race before they finally stop.

These observations show that bodies have an in-built reluctance to changes in their state of motion or rest. The tendency of a body to resist change in its state of rest or motion is called **inertia**. Hence Newton's first law of motion is also referred to as the **law of inertia**.

4.3: Newton's second law

This law states: the rate of change of momentum of a body is directly proportional to the resultant external force acting on the body and takes place in the direction of the force.

Moment of a body is defined as the product of its mass and velocity. Since velocity is a vector quantity, momentum is also a vector quantity having both magnitude (size) and direction.

Momentum P=mass m*velocity v

Hence the unit of momentum is the kilogram-metre per second (kgm/s). The direction of momentum is the same as that of the velocity. The change of momentum is therefore caused by a change in velocity.

Suppose the velocity of a body of mass **m** changes from an initial value **u** to a value **v** after a time **t**, then:

The initial momentum P_i=mu

The final momentum P_f=mv

The change in momentum= final momentum- initial momentum

Thus $\Delta P= P_f - P_i$= mv- mu=m(v-u)

Therefore, the rate of change of momentum= ΔP/t = m(v-u)/t.

From the equations of linear motion, (v-u)/t =acceleration **a**

Hence ΔP/t =ma.

From the second law of motion, Fαma.

And so the force F= mass m*acceleration a (F=ma).

Therefore, F=ma=m(v-u)/t

And Ft=m(v-u).

The product of the force and time is called **impulse**. It is a vector quantity since force is a vector quantity. The unit of impulse is the newton-second(Ns). Impulse is also equal to the change in momentum(mv-mu). Hence impulse can also be expressed in kgm/s.

Example 2.1

1. Two stones of mass 8kg and 4kg move with velocities 3m/s and 6m/s respectively. Compare their momentum.

P_{8kg}=mv =8*3=24kgm/s

P_{4kg}=mv =4*6=24kgm/s

Hence they have the same momentum.

2. A ball of mass 35g travelling horizontally at 20m/s strikes a barrier normally and rebounds with a speed of 3m/s. Find the impulse exerted on the ball.

   Impulse=Ft=m(v-u)= (0.035*20) – (0.035*-3)

   =1.26Ns

   Note that the two speeds are in opposite directions.

3. A kick that lasts 0.03s sends a ball of mass 0.65kg with a velocity of 15m/s northwards. Find:

a) The change in momentum of the ball.

   Note that the ball is initially at rest, i.e. u=0m/s.

   ΔP=mv-mu=(0.65*15)-(0.65*0)=9.75kgm/s

b) The average force exerted on the ball.

   F=m(v-u)/t =(9.75kgm/s)/0.03s)=325N

c) The displacement of the ball in 2 seconds.

   The upward acceleration of the ball is negative $2m/s^{5.}$

$S=ut+1/2at^2=(15*0.03)+(1/2*-2*0.03^2)=2m/s.$

2.4: Newton's third law

The law states: for every action there is an equal and opposite reaction. We look at the working of a lift in relation to the third law of motion in three situations:

a) **When the lift is at rest.**

This implies that the resultant force on the lift is zero i.e. action and reaction are equal in size. The force acting on the lift is the weight of the person standing in the lift. This is balanced by the reaction by the floor of the lift.

Therefore, weight mg=- reaction R,

Or simply; mg+R=0.

b) **When the lift descends with an acceleration a**

For the lift to move downwards, the weight of the occupant must be greater than the reaction by the floor of the lift. Therefore, the resultant force pulling the lift downwards is equal to the difference between the weight mg and the reaction R;

Resultant force F= mg-R.

From the second law of motion, the resultant force F=ma.

Therefore, ma=mg-R.

And R=mg-ma =m(g-a).

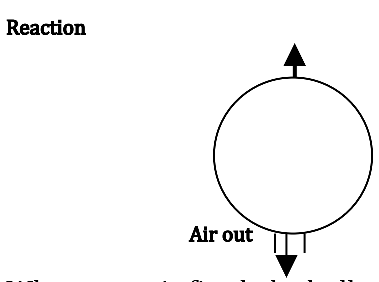
c) **When the lift ascends with an acceleration a**

In this case, the reaction by the floor of the lift must be greater than the weight of the occupant. Hence, the resultant force F=ma=R-mg.

And R=ma+mg=m(a+g).

The following are some cases where the third law of motion has been applied in everyday life:

- A balloon moves in an opposite direction when air in it is released.



- When a gun is fired, the bullet leaves the gun while the gun recoils backwards.
- For a person running or walking, one exerts a backward force on the ground with the ground exerting a forward push on the foot of the person. This makes running or walking possible.

Example 2.2

1. A man of mass 75kg stands on a weighing machine in a lift. Determine the reading on the weighing machine when the lift:

a) Ascends with an acceleration of $2m/s^{5.}$

F=ma=R-mg

(75*2)=R-(75*2)

R=150+750 =900N

b) Descends at a constant velocity of 1.5m/s.

F=ma=mg-R

But a=0 since the velocity is constant.

Therefore, 75*0=75*2 – R

R=750N

c) Descends with an acceleration of $5.5m/s^{5.}$

75*5.5= 75*2 – R

R=750 - 183.5=565.5N

2. A car of mass 1500kg is brought to rest from a velocity of 25m/s by a constant force of 3000N. Determine the change in momentum produced by the force and the time it takes the car to come to rest.

ΔP=mv-mu=1500(0-25)=-37500kgm/s.

Ft=ΔP

We ignore the negative sign in this part because time is a scalar quantity.

3000*t=37500

t=37500/3000 =15.5seconds.

2.5: Collision and the law of conservation of momentum

This body states that when two or more bodies collide, their total linear momentum before and after collision remain constant provided no external force acts on them;

i.e. momentum before collision= momentum after collision.

There are basically two types of collisions namely elastic and inelastic collision.

a) Elastic collision

This is where the bodies move separate ways after collision. In this collision, not only linear momentum is conserved but also kinetic energy;

- Total linear momentum before collision= total linear momentum after momentum.

- Total kinetic energy before collision= total kinetic energy after collision.

b) Inelastic collision

This is where the colliding bodies stick together and move as one body after collision. In this type of collision, it is only linear momentum which is conserved but not kinetic energy. This is because during this collision, some deformation takes place which eats up part of the energy while some is converted to heat, sound or light energy.

- Total linear momentum before collision= total linear momentum after collision.

Example 2.3

1. A bullet of mass 20g is shot from a gun of mass 20kg with a muzzle velocity of 200m/s. if the bullet is 30cm long, determine:

a) The acceleration of the bullet.

For the bullet: u=0, v=200m/s, s=0.3m

$v^2=u^2+2as$

$200^2=0+(2)(0.3a)$

a=40000/0.6 =$5.667*2^4 m/s^2$

b) The recoil velocity of the gun.

Total linear momentum before collision=total linear momentum after collision

(20*0)+(0.02*0)=(20*v)+(0.02*200)

v=-4/20= -0.2m/s.

2. A 5kg mass moving with a velocity of 2m/s collides with a 2kg mass moving at 7m/s along the same line. If the two masses join together on impact, find their common velocity if they were moving:

a) In opposite directions.

Total linear momentum before collision=total linear momentum after collision

(5*2) + (2*-7)=(5+2)v

15v=-20

v=-20/15 =-1.33m/s

the bodies move in the initial direction of the 2kg mass.

b) In the same direction.

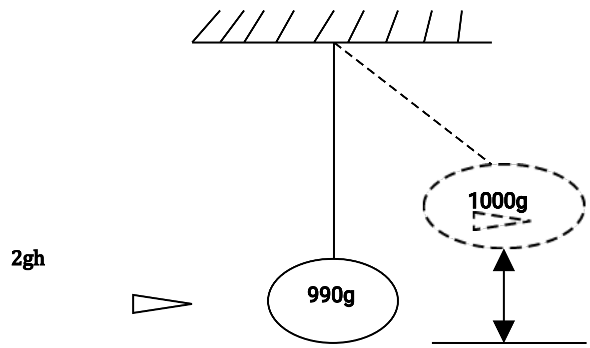
Total linear momentum before collision=total linear momentum after collision

(5*2)+(2*7)=(5+2)v

15v=120

v=120/15 =8m/s

3. A bullet of mass 2g travelling horizontally at 20m/s embeds itself in a block of wood of mass 990g suspended from a light inextensible string so that it can swing freely. Find:



a) The velocity of the bullet and block immediately after collision.

(0.01*20)+(0.99*0)=(0.01+0.99)v

v=1/1 =1m/s

b) The height through which the block rises.

At the maximum height, all the kinetic energy is converted into potential energy.

k.e=p.e

½(mv^2)=mgh

½(0.01+0.99)1^2=(0.01+0.99)(2)h

h=0.05m

2.6: Friction

This is a force acting between two surfaces in contact and tends to oppose the intended motion. Friction may be beneficial but can also be a nuisance.

2.6.1: Advantages of friction

- Makes walking, writing possible.

- Required for braking in cars, bicycles etc.
- Makes rotation of the conveyor belts in factories possible.
- Necessary for lighting matchsticks.
- Useful when using nuts, bolts, screw jacks, vices etc.

2.6.2: Limitations of friction

- A lot of energy is lost in the form of heat.

- Causes wear and tear on the pars of machines.

- May lead to noise pollution.

It is therefore important to minimize friction at all cost. This can be done through the following ways:

- Using rollers.
- Using ball bearings.
- Lubrication
- Air cushioning.

2.6.3: Factors affecting friction

Frictional force is directly proportional to the normal reaction R;

$F \alpha R$

Or simply F/R= a constant.

The constant is called coefficient of friction μ. It is a measure of the nature of the surfaces in contact.

Hence, frictional force F= normal reaction R* coefficient of friction μ.

When the two bodies are at rest, then the coefficient of friction is referred to as **coefficient of static** friction while if they are in relative motion, it is called **coefficient of kinetic** friction. Coefficient of friction has no units.

Hence, friction depends on two factors:

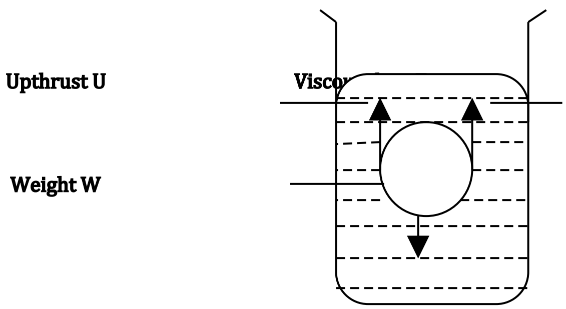
1. The normal reaction R.
2. The nature of the surface. Frictional force is greater between rough surfaces than between smooth surfaces.

Note that frictional force is independent of the area of contact of the two surfaces and relative velocity of the bodies.

2.7: Viscosity

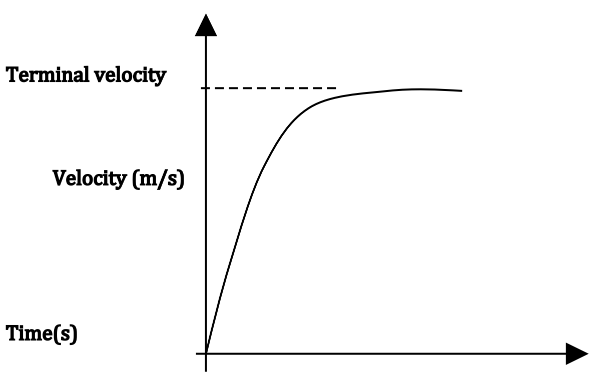
Friction exerted by fluids is called **viscosity** or **viscous drag**. It is the force which opposes relative motion between layers of the fluid. Viscosity is caused by the forces of attraction between the molecules of the fluid. When a body is put in a fluid, three forces act on it, namely:

- Weight of the body which acts downwards.
- Upthrust due to the fluid which acts upwards.
- Viscous drag due to the fluid which acts upwards.



When the body enters the fluid, its weight is initially higher than the total upward forces i.e. upthrust plus viscous drag. The resultant force acting on the body accelerates it towards the bottom of the container. As the body sinks down, the viscous drag increases until the three forces balance i.e. W= U+ F. at this point, the body attains its maximum constant velocity called **terminal velocity**. The resultant force on the body is therefore zero.

The graph of velocity against time for a body falling through a fluid appears as shown below:



Note that viscosity decreases with increase in temperature.

TOPIC 3.: WORK, ENERGY, POWER AND MACHINES

3.1: Work and Energy

When a force acting on a body displaces the body in the direction of the force **work** is said to have been done. Work is the product of force and displacement in the direction of the force;

Workdone= force F*displacement s.

The SI Unit of work is newton-metre (Nm).

1Nm= 1joule (1J).

A joule is defined as the workdone by a force of one newton to displace a body through one metre in the direction of the force.

Other multiples of the joule include kilojoule(kJ) and megajoule(MJ).

Energy on the hand is the ability or capacity to do work. Anything that possesses energy is capable of doing work. The SI Unit of energy is the joule. Energy has the following characteristics:

- It is not visible.
- Occupies no space.
- Has no mass nor any other physical property.

The most common sources of energy include the sun, wind, geothermal, waterfalls, nuclear or atomic energy, fuels etc.

Energy resources may be grouped into two:

- Renewable energy- can be reused again and again. Their supplies are inexhaustible e.g solar, geothermal, wind energy.
- Non-renewable energy- their supplies are exhaustible i.e. cannot be reused once exhausted e.g. wood, coal biogas, petroleum etc.

Energy exists in many forms such as mechanical, chemical, heat and electrical energy amongst others. In this topic, we will look at mechanical energy.

3.1.1: Mechanical energy

It is divided into two areas namely kinetic energy and potential energy.

Kinetic energy is the energy possessed by a body in motion. Suppose a body of mass m is moving with a constant velocity v, then its kinetic energy is given by;

Kinetic energy=½(mv^2).

Potential energy on the other hand is a form of stored energy in a body when it is in a particular state or position. A body in a raised position possesses **gravitational potential energy** given by;

$P.E_g$=mgh, where m- mass of the body, g- gravitational field strength and h- height above the ground.

Also, a stretched or compressed material is able to regain its original shape when released. This is because it possesses a type of potential energy known as **elastic potential energy**. As can be recalled from Hooke's law, the workdone in stretching or compressing an elastic material is given by;

W=½(Fe) =½(ke^2).

Hence the elastic potential energy is given by;

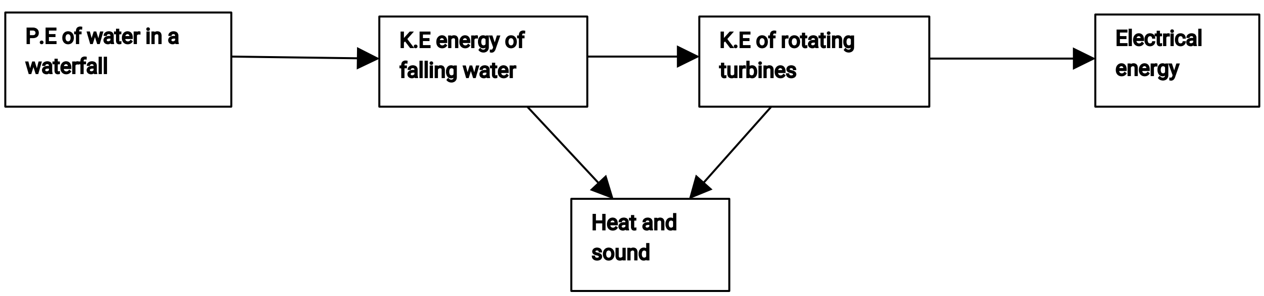
$P.E_e$=½(Fe) =½(ke^2).

3.1.2: The law of conservation of energy

The law states: energy can neither be created nor destroyed but can be transformed from one form to another.

Alternative statement: the sum of kinetic energy and potential energy of a system is a constant.

Below is the energy transformation in a hydroelectric power station:



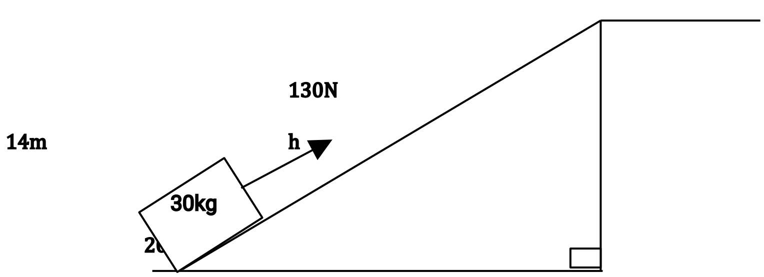
Example 3.1

1. A force of 40N is applied on a body. The body moves a horizontal distance of 7m. Calculate the workdone on the body.

   W=F*s =40N*7m

   =280Nm or 280J

2. A box of mass 30kg is pushed up an inclined plane of length 14m using a force of 130N as shown below:



If the track is inclined at an angle of 20^0, calculate:

a) The height of the platform.

Sin 20^0=h/14

h=14sin 20^0=

b) Workdone by the force of 130N.

W=F*s = 130*14 =1820J

c) Workdone, if the box is lifted vertically upwards. Compare your answer in (b) and (c) above.

W=mgh=300sin 20^0=

Workdone in pushing the body along the inclined plane is greater than the workdone when lifting the body vertically upwards. This is because of the frictional force between the body and the inclined plane.

d) The frictional force between the box and the inclined plane.

F_r=1820-300sin 20^0=

3. A crane is used to lift a body of mass 30kg through a vertical distance of 5.0m.

a) How much work is done on the body?

W=F*s =(mg)s= 300*6=1800J

b) What is the potential energy stored in the body?

P.E=mgh=30*2*6=1800J

c) Comment on the two answers above.

Workdone on the body is equal to the potential energy stored in the body. Hence the workdone against gravity is stored as the potential energy.

4. A spring of spring constant 25N/m is stretched such that its length increases from 2cm to 20cm. calculate the amount of workdone on stretching the spring.

W=½(ke^2)= ½(25)(0.1^2).

=0.125J

5. A body of mass 12kg is pulled from the rest with a constant force of 25N. The force is applied for 5.0s. Calculate:

a) The distance travelled.

F=ma

a=25N/12kg =5.1m/s^2, u=0, t=6

s=ut+1/2at^2=(0*6)+1/2(5.1)(6^2)=33.8m

b) Workdone on the body.

W=F*s =25*33.8 =945J

c) The final kinetic energy of the body.

K.E=workdone= 945J

d) The final velocity of the body.

$K.E=1/2(mv^2)=945J$

$v=\{(2*945)/12\}^{1/2}= 15.6m/s.$

3.2: Power

Power is defined as the rate of doing work;

Power=workdone/time.

The SI Unit of power is the watt (W).

1W= 1J/s.

Other multiples of the watt include the kilowatt(kW) and megawatt(MW);

$1W=2^{-3}kW$

$1W=2^{-6}MW$

The power of a device is the measure of how fast the device can perform a given task or convert a given amount of energy. For example, a device rated 1kW converts 200J of energy to another form in one second.

Power=workdone/time =Fd/t.

But d/t =velocity v.

Therefore, power= force F*velocity v.

Example 3.2

1. A person of mass 60kg climbs 3m up a rope in 20seconds. Find the average power developed by the person.

Power=workdone/time =(600*3)/20 =480W

2. A person of mass 40kg runs up a flight of 50stairs each of height 20cm in 5 seconds. Calculate:

a) The workdone.

W=mgh=40*2*(50*0.2)=4000J

b) The average power of the person.

Power=4000J/5s =800W

c) Explain why the energy the person actually uses to climb up is greater than the calculated workdone.

3.3: Machines

A machine is a device that makes work easier. In a machine, a force applied at one point of a system is used to generate another force at a different point of the system to overcome a load. The following terms are used in machines:

a) **Effort-** the force applied to the machine.

b) **Load-** the force exerted by the machine.

c) **Mechanical advantage (M.A)-** the ratio of the load to effort.

M.A=Load/Effort.

It has no units.

It is dependent on friction between the moving parts and the weight of the parts of the machine that have to be lifted when operating the machine; the greater the friction the smaller the mechanical advantage.

d) **Velocity ratio (V.R)**- it is defined as the ratio of the velocity of the effort to the velocity of the load;

V.**R**= velocity of effort/velocity of load = $\frac{\text{Effort distance/time}}{\text{Load distance/time}}$

Thus V.R=effort distance/load distance.

Velocity ratio also has no units.

e) **Efficiency η**

It is the ratio of the workdone on the load (work output) to the workdone by the effort (work input) expressed as a percentage;

Efficiency η= (work output/work input)*5.

Efficiency also depends on the friction between the moving parts and the weight of the moveable parts. Hence the efficiency of a machine is always less than 20%.

Efficiency=work output/work input= (load*load distance)/ (effort*effort distance)

= (load/effort)*(load distance/effort distance)

But load/effort =mechanical advantage (M.A),

And, load distance/effort distance =1/velocity ratio

Therefore, efficiency η= (M.A/V.R)*5.

Example 3.3

1. A machine requires 6000J of energy to lift a mass of 55kg through a vertical distance of 8m. Calculate its efficiency.

   Work input= 6000J

   Work output=F*s= 55*2*8 =4400J

   Efficiency=(work output/work input)*20= (4400/6000)*20 = 73.33%

2. An effort of 250N raises a load of 900N through 5m in a machine. If the effort moves through 25m, find:

a) The useful workdone in raising the load.

Useful workdone=load*load distance =900*5= 4500J

b) The workdone by the effort.

Workdone by the effort= effort*effort distance= 250*25= 6250J

c) The efficiency of the machine.

Efficiency= (work ouput/work input)*20= (4500/6250)*20 = 72%.

3. A machine whose velocity ratio is 8 is used to lift a load of 300N. The effort required is 60N. calculate:

a) The mechanical advantage of the machine.

M.A= load/effort =300/60 =5

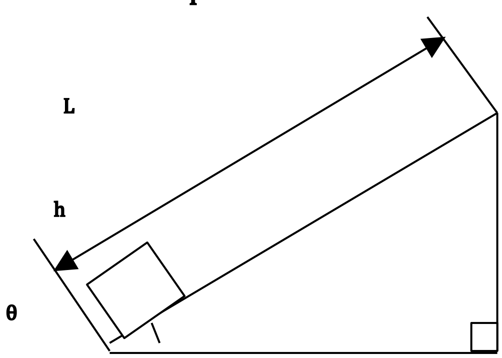
b) The efficiency of the machine.

Efficiency= (M.A/V.R)*20= (5/8)*20 =65.5%

3.4: Types of machines

Below are some of the common machines:

3.1.1: Inclined plane



The distance moved by the effort is L while the vertical height moved by the load is h.

Also, sin θ=h/L

Or simply h=Lsinθ

Therefore, velocity ratio (V.R)= effort distance/ load distance =L/Lsin θ.

Hence V.R= 1/sin θ.

Example 3.4

1. A man uses an inclined plane to lift a 81kg mass through a vertical height of 1.0m. Given that the angle of inclination of the plane is 30^0 and its efficiency is 75%, determine:

a) The effort needed to move the load up the inclined plane at a constant velocity.

V.R=1/sin 30 =2

Therefore, (M.A/2)*20=75

M.A= (2*75)/20 =3/2

3/2 = 82N/effort

Effort= (82*2)/3 =540N

b) The workdone against friction in raising the mass through the height of 1.0m.

Work input=effort*effort distance = (540*4)/sin 30 =4320J

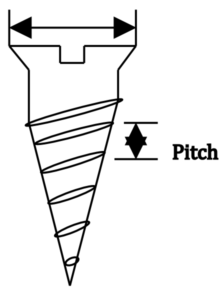
Work output=load*load distance= 81*2*4= 3240J

Therefore, workdone against friction= 4320-3240= 1180J

3.1.2: A screw and bolt

For a screw, when the effort applied on the head moves through a complete revolution, the screw advances by a distance equivalent to one pitch. A pitch is the distance between two successive threads.

d

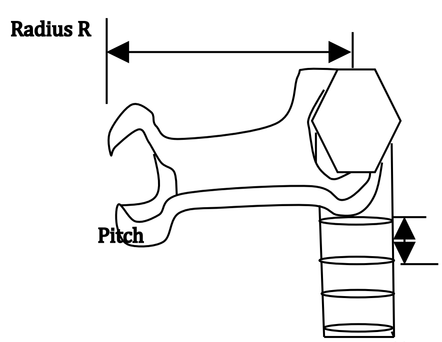


Distance moved by the effort= circumference =πd

Distance moved by the load= one pitch

Hence, velocity ratio (V.R)= circumference/pitch =πd/pitch.

For the bolt, effort is applied at the free end of the spanner.



Therefore, the distance moved by the effort in one revolution= circumference= 2πR.

Hence, V.R= circumference/pitch =2πR/pitch.

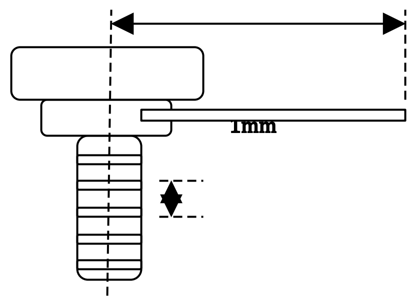
Note that a combination of a screw and lever can be used as a jack for fitting heavy loads e.g car jack. When two or more systems are combined together, the overall velocity ratio is the product of the individual velocity ratios;

Combined V.R= V.R_1*V.R_2*………..*V.R_k

Example 3.5

1. The figure below shows a screw jack whose screw has a pitch of 1mm and has a handle of 25cm long.

25cm



Determine the velocity ratio of the jack.

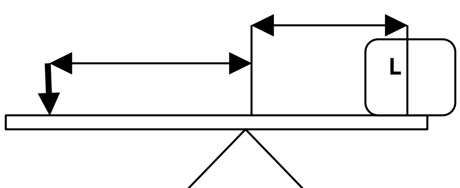
V.R= 2πr/pitch= 2π(25cm)/0.1cm= 1571

3.1.3: Lever system

Load arm

Effort arm

Effort



The velocity ratio of a lever system is the ratio of the effort arm to the load arm;

V.R= Effort arm/ Load arm.

3.1.4: Gears

A gear is a wheel with equally spaced teeth or cogs around it. The wheel on which the effort is applied is called the driving (input) gear while the load gear is referred to as the driven (output) gear. Suppose the driving gear has **n** teeth and the driven gear**N** teeth, then when the driving gear makes one complete revolution the driven gear makes **n/N** revolutions.

V.R of the system = Number of revolutions made by the effort (driving) gear / Number of revolutions made by the load (driven) gear.

V.R = 1revolution / n/N revolutions =N/n

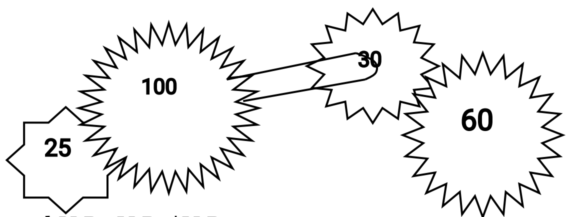
Hence, velocity ratio of a gear system is the ratio of the number of teeth of the driven gear to the number of teeth of the driving gear;

$$V.R = \frac{\text{Number of teeth of the driven gear}}{\text{Number of teeth of the driving gear}}$$

Example 3.6

1. A driving gear having 25teeth engages with a second gear with 20teeth. A third gear with 30 teeth on the same shaft as the second one engages with a fourth gear having 60teeth. Find:

a) The total velocity ratio of the system.



Combined $V.R = V.R_1 * V.R_2$

$V.R_1$=No. of teeth of driven gear/ No. of teeth of driving gear

= 20/25= 4

$V.R_2$=60/30= 2

Hence, V.R= 4*2= 8

b) The mechanical advantage of the system if its efficiency is 85%.

Efficiency= (M.A/8)*20= 85

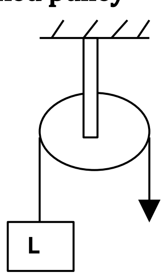
M.A= (85*8)/20= 5.8

3.1.5: Pulleys

A pulley is a wheel with a groove to accommodate a string or rope. There are three possible systems of pulleys namely single fixed, single moveable and a block and tackle.

a) Single fixed pulley

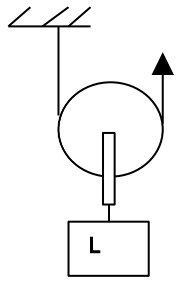
E



In this arrangement, both the effort and load move through the same distance. Hence the velocity ratio of the system is one.

b) Single moveable pulley

E



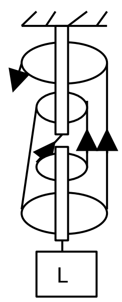
The load is supported by two sections of the string. If the load is pulled upwards through a distance of 1m, each section of the string also moves through 1m. Hence the effort moves through a total distance of 2m.

Therefore, the velocity ratio of the system = effort distance/load distance =2m/1m =5.

c) A block and tackle

This system comprises two sets; one set fixed and the other moveable. A single string is then passed around each pulley in turn. The arrangement can take several forms depending on the desired velocity ratio. Below is an example:

E



In this case, there are four sections of the string supporting the load. Hence, when the load moves upwards through a distance of 1m, each section of the string also shortens by 1m. Therefore, the total distance moved by the effort (string) is 4m.

Thus, V.R of the system= effort distance/load distance =4m/1m =1. Coincidentally, the velocity ratio of the system is the same as the number of sections of the string supporting the load.

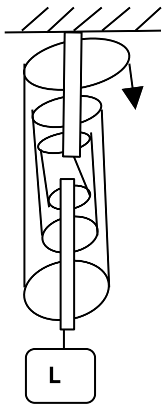
Generally, the velocity ratio of a block and tackle system is given by the number of sections of the string supporting the load.

Practically, the efficiency of any pulley system is less than 20%. This is as a result of two reasons:

- The friction between the moveable parts.
- The weight of the parts that have to be lifted when operating the system.

Example 3.7

1. The figure below shows a pulley system used to raise a load.



a) State the velocity ratio of the system.

V.R=number of strings supporting the load= 6

b) If an effort of 200N is needed to raise a load of 4500N, determine the efficiency of the system.

M.A= load/effort= 4500N/200N= 1.5

Efficiency = (M.A/V.R)*20= (1.5/6)*20= 75%

c) Calculate the wasted energy if a mass of 500kg is lifted up through a height of 2m using the same system.

Work ouput= load*load distance= 500*2*2= 2000J

Efficiency= (work output/work input)*20

Therefore, (2000J/work input)*20=75

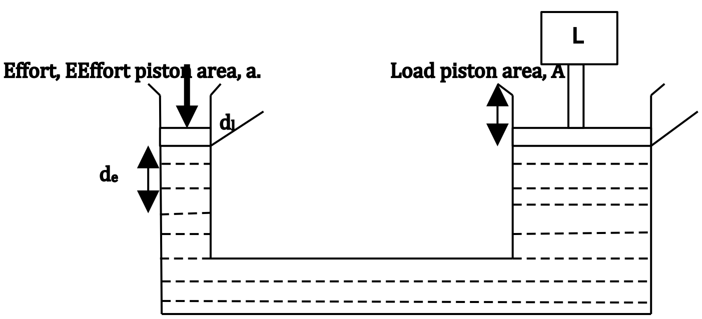
Work input= (2000*20)/75 =13333.33J

Wasted energy= 13333.33-2000= 3333.33J

Alternatively, wasted energy=25% of work input= (25/20)*13333.33J=3333.33

3.6: Hydraulic machine

Consider the diagram below:



When the effort is applied as shown, the volume of the liquid leaving the effort arm is the same as the volume of the liquid entering the load arm;

i.e. $a*d_e=A*d_l$,

$d_e/d_l= A/a$

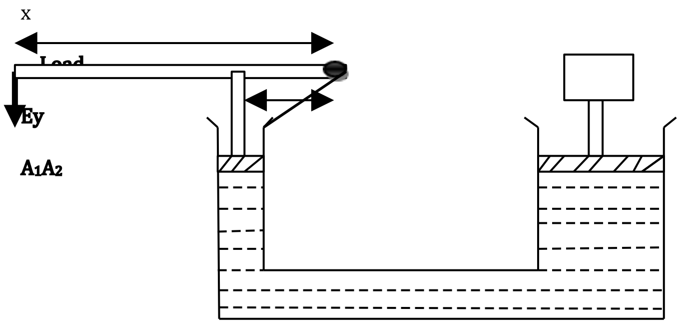
Therefore, the velocity ratio of a hydraulic system is the ratio of the area of the load piston to the area of the effort piston. If the pistons are circular then;

V.R=area of load piston/area of effort piston $=\pi R^2/\pi r^2$

V. $=R^2/r^2$, where R- is the radius of the load piston and r- is the radius of the effort piston.

Example 3.8

1. In the figure below x=30cm, y=6cm, effort E=60N, A_1=4cm^2 and A_2=12cm$^{5.}$



Calculate:

a) The force F exerted on the liquid at A_1.

By the principle of moments;

60N*30cm= F*6cm

F= (60*30)/6= 300N

b) The velocity ratio of the system.

V.R of the lever system= effort arm/load arm =30cm/6cm= 5

V.R of the hydraulic system= area of load piston/area of effort piston= 12cm^2/4cm^2= 3

Therefore, the combined V.R= 5*3= 15

c) The maximum load that can be raised by the system.

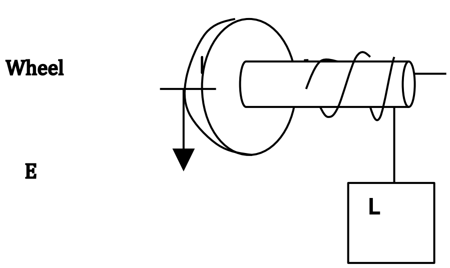
Pressure at A_1= Pressure at A_2

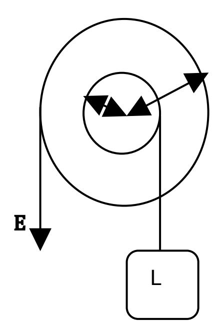
300N/4cm^2 =L/12cm^2

L= (300*12)/4 =900N.

3.7: Wheel and axle

It consists of a large wheel of radius R attached to an axle of radius r.





Note that in this case, both the wheel and axle make the same number of revolutions at any time;

Thus, in one revolution the distance moved by the effort= 2πR,

And the distance moved by the load= 2πr.

Hence, the velocity ratio of the system= 2πR/2πr = R/r.

Thus the velocity ratio of a wheel and axle is the ratio of the radius of the wheel to the radius of the axle.

Example 3.9

1. A wheel and axle is used to raise a load of 140N by a force of 20N applied to the brim of the wheel. If the radii of the wheel and axle are 70cm and 5cm respectively, calculate the mechanical advantage, velocity ratio and efficiency of the system.

   M.A= load/effort =140N/20N= 7

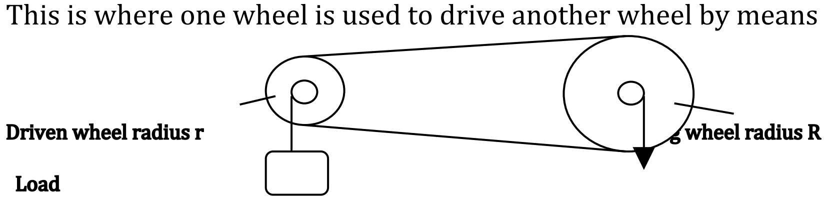
   V.R =radius of the wheel/radius of the axle= 70cm/5cm= 14

   Efficiency= (M.A/V.R)*20

   = (7/14)*20= 50%

3.8: Pulley belt

This is where one wheel is used to drive another wheel by means of a belt.



The driving wheel covers a distance 2πR in one revolution while the driven wheel covers a distance 2πr in one revolution. If the driving wheel makes one revolution, the driven wheel makes 2πR/2πr (R/r) revolutions.

V.R of the system= $\frac{\text{Number of revolutions made by the effort (driving) wheel}}{\text{Number of revolutions made by the load (driven) wheel}}$

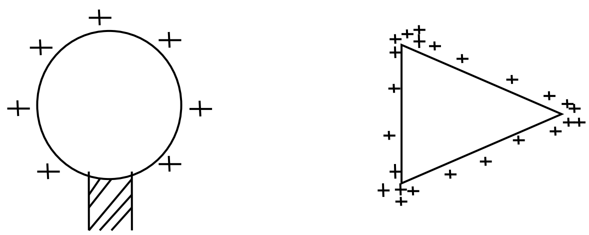
V.R = 1/(R/r) =r/R

Therefore, the velocity ratio of a pulley belt is the ratio of the radius of the driven (load) wheel to the radius of the driving (effort) wheel.

4. LECTROSTATICS II

4.0 Charge distribution on the surface of a conductor

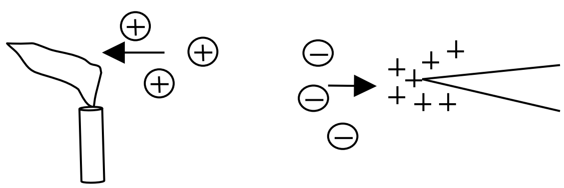
The quantity of charge per unit area of the surface of a conductor is called **charge density.** The charge distribution on a conductor depends on the shape of the conductor. Generally, the charge concentration on a spherical conductor is uniform while that on a sharp point is high.



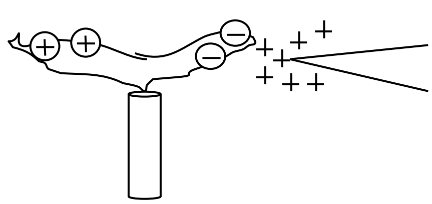
The high charge concentration at sharp points makes it easier to gain or lose charges. The effects of high charge concentration at sharp points can be seen in the following cases:

Electric wind

When a highly charged sharp point is brought close to a candle flame, the flame is observed to drift away as if there was wind. The high charge concentration at the sharp point ionizes the surrounding air producing both positive and negative charges. Opposite charges are attracted to the point while similar charges are repelled away from the point blowing away the flame.



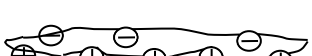
If the point is brought very close, the flame splits into two; one part moves towards the point and the other part away from the point. This is because a flame has both positive and negative ions. The negative ions are attracted towards the point while the positive ions are repelled away from the point.



Lightning arrestors

When clouds move in the atmosphere, they rub against the air particles and produce a large amount of static charges by friction. These charges induce large amounts of the opposite charge on the earth. Hence a high potential difference is created between the earth and cloud. This makes air to be a charge conductor. The opposite charges attract each other and neutralize, causing thunder and lightning. Lightning can be very destructive to buildings and other structures.

Lightning arrestors are used to safeguard such structures. It consists of a thick copper plate buried deep under the ground. The plate is connected by a thick copper wire to the spikes at the top of the building. The arrestor assumes the same charge as the earth. At the spikes, a high charge density builds up and a strong electric field develops



between the cloud and the spikes. The air around the spikes is ionized. The opposite charges attract each other and neutralize. Excess electrons flow to the ground through the thick copper wire.

It is for this reason that people are advised not to take shelter under trees when it is raining.

Applications of static charges

- **Electrostatic precipitator**

One of the causes of air pollution globally is increased industrialization. Some industries have indeed responded to this challenge by installing electrostatic precipitators which are found within the chimneys.

An electrostatic precipitator consists of a cylindrical metal plate fixed along the walls of the chimney and a wire mesh suspended through the middle.

The plate is charged positively by connecting it to a high voltage, approximately 50,000V and the wire mesh charged negatively. As a result, a strong electric field exists between the plate and the wire mesh. The ionized pollutant particles get attracted; some to the plate and others to the wire mesh.The deposits are removed occasionally. The same principle is used in fingerprinting and photocopying.

- **Spray painting**

The nozzle of the spraying can is charged. When spraying, the paint droplets acquire similar charge and spread out finely due to repulsion. As the droplets approach a metallic body, they induce opposite charge which then attracts them to the metal surface. This ensures that little paint is used.

Dangers of static charges

When a liquid flows through a pipe, its molecules rub against each other and against the walls of the pipe and become charged. If the liquid is flammable like petrol, it is likely to cause sparks or even explosion. This can also happen to fuels when they are packed in plastic containers.

It is therefore advisable to store fuels and other flammable liquids in metallic containers so that any charges generated can continually leak out. This also explains why long chains hang underneath fuel tankers as they move.

4.1: Electric field

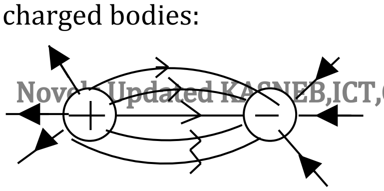
This is the region around a charged body where its influence (attraction and repulsion) can be felt. It is represented lines of force called electric field lines. The direction of an electric field is the direction in which a positive charge would move if placed at that point.

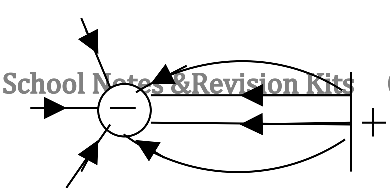
Electric field lines have the following properties:

- Originate from a positive charge and terminate at a negative charge
- Do not cross each other i.e. do not intersect
- Are parallel at uniform field, close together at strong fields and widely spaced at weaker fields.

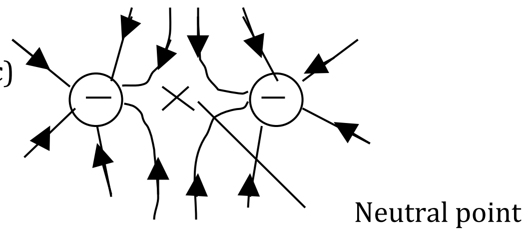
4.2: Electric field patterns

The electric field pattern between two charged bodies obeys the law of electrostatics. Below are some patterns between charged bodies:





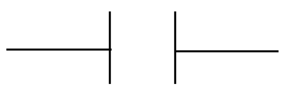
(a) (b)

c) 

NB/At the neutral point, the resultant effect is zero.

4.3: Capacitors

A capacitor is a device used for storing charge. It consists of two or more metal plates separated by a vacuum or a material medium (insulator). This material is known as a 'dielectric'. Other materials that can be used as a dielectric include air, plastic, glass e.t.c. the symbol of a capacitor is shown below:

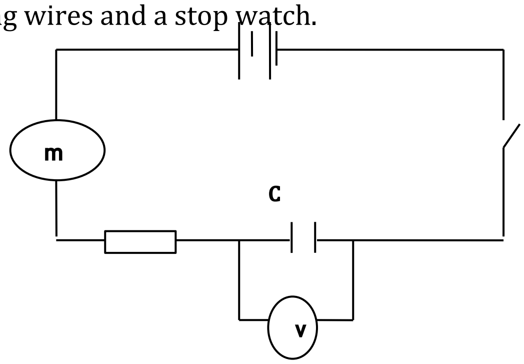


There are three main types of capacitors namely paper capacitors, electrolytic capacitors and variable capacitors. Others include plastic, ceramic and mica capacitors.

4.4: Charging a capacitor

Experiment: To charge a capacitor

Apparatus : Uncharged capacitor of 500μF, 5.0V power supply, rheostat, voltmeter, milliammeter, switch, connecting wires and a stop watch.



Procedure

i. Set up the apparatus as shown above.

ii. Close the switch and record the values of current, I at various time intervals. Tabulate your values in the table below:

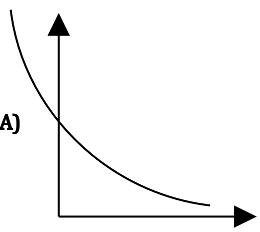
Time, t(s)	0	2	20	30	40	50	60	70
Current, I(mA)								
It (mAs)								

iii. Plot a graph of current, I against time, t

iv. Plot a graph of It against time.

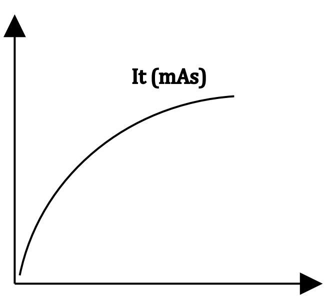
Observations

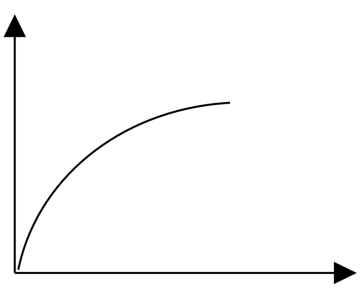
The charging current is initially high but gradually reduces to zero. A graph of current, I against time appears as shown below:



The charging current drops to zero when the capacitor is fully charged. As the p.d. across the capacitor increases the charge in the capacitor also increases up to a certain value. When the capacitor is fully charged, the p.d across the capacitor will be equals the p.d of the source.

A graph of p.d across the capacitor against time is exponential. A graph of It against time is also exponential.





NB

The product It represents the amount of charge in the capacitor.

1.5: Discharging a capacitor

Experiment: To discharge a capacitor

Apparatus :A charged capacitor, resistor, galvanometer, switch and connecting wires.

Procedure

- Set up the apparatus as shown above.
- Close the switch and record the values of current at various time intervals in the table below.

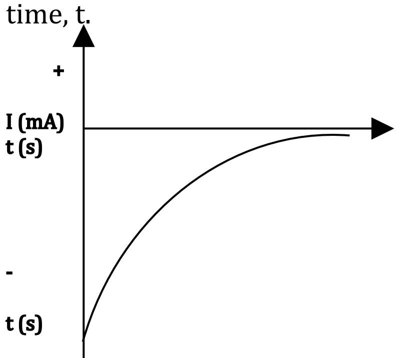
Time, t(s)	0	2	20	30	40	50	60	70
Current, I (mA)								

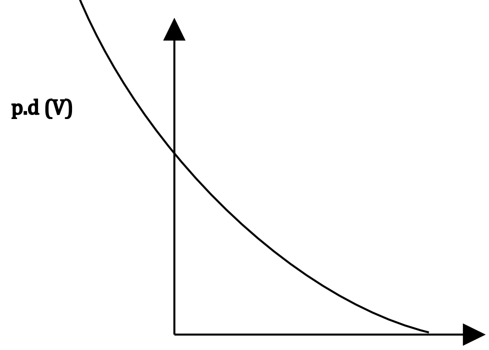
- Plot a graph of current, I against time, t.

Observations

The value of current is seen to reduce from maximum value to zero when the capacitor is fully discharged. The galvanometer deflects but in the opposite direction to that during charging.

During discharging, the p.d. across the capacitor reduces to zero when the capacitor is fully discharged. The graphs below show the variation between current, I and time, t and between the p.d across the capacitor and time, t.





A graph of charge in the capacitor, Q against time, t during discharging also appears like that of p.d against time i.e. p.d across the capacitor is directly proportional to the charge stored.

4.6: Capacitance

Capacitance of a capacitor is defined as the measure of the charge stored by the capacitor per unit voltage; C = Q/V

Hence Q = CV

Recall: Q = It

Therefore Q= CV = It

The SI Unit of capacitance is the farad, F. A farad is the capacitance of a body if a charge of one coulomb raises its potential by one volt.

Other smaller units of capacitance are: microfarad (μF), nanofarad (nF) and picofarad (Pf).

i.e. 1 μF = 2^{-6} F

1 nF = 2^{-9}F

1 pF = 2^{-12}F

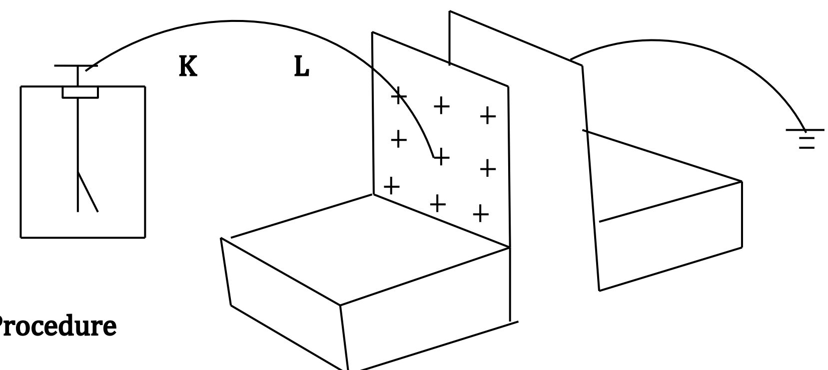
4.7: Factors affecting capacitance of a capacitor

The capacitance of a parallel plate capacitor depends on three factors, namely:

- Area of overlap of the plates, A
- Distance of separation, d between the plates
- Nature of the dielectric material

Experiment: To investigate the factors affecting capacitance

Apparatus: 2 aluminium plates, K and L of dimensions 25cm * 25cm**,**Insulating polythene support**,** uncharged electroscope**,** Glass plate**,** earthing wireand a free wire.



Procedure

- Fix the plates on the insulating support so that they stand parallel and close to each other as shown above.
- Charge plate K to a high voltage and then connect it to the uncharged electroscope. Earth the second plate, L.
- While keeping the area of overlap, A the same vary the distance of separation, d and observe the leaf divergence.
- While keeping the distance of separation, d constant vary the area of overlap, A and observe the leaf divergence.
- While keeping both the area of overlap and the distance of separation, d constant introduce the glass plate between the plates of the capacitor and observe what happens to the leaf.

Observations

1. When the distance of separation is increased the leaf divergence also increased.
2. When the area of overlap is increased the leaf divergence decreased.
3. When the glass plate is introduced between the plates, the leaf divergence increased.

Note that the leaf divergence here is a measure of the potential, V of plate K. Hence the larger the divergence the greater the potential and thus the lower the capacitance (since C = Q/V, but Q is constant).

Conclusion

From the above observations, it follows that the capacitance is directly proportional to the area of overlap between the plates and inversely proportional to the distance of separation. It also depends on the nature of the dielectric material.

$C \propto A/d$

$C = \varepsilon A/d$ where ε is a constant called permittivity of the dielectric material (epsilon).

If between the plates is a vacuum, then $\varepsilon = \varepsilon_0$, known as epsilon nought and is given by $6.85 * 2^{-12}$ Fm^{-1}. Hence $\mathbf{C = \varepsilon_0 A/d}$

Example 9.1

1. How much charge is stored by a 300μF capacitor charged up to 12V? give your answer in (a) μC (b) C {**ans. 3600μC/0.0036C**}

   Solution

   a) Q= CV = 300 * 12 =3600μC b) 3600 * 2-6 =0.0036C

2. What is the average current that flows when a 720μF capacitor is charged to 2V in 0.03s?

   {ans. 0.24A}

   Solution

   Q = CV =It

   I= 720 * 2-6 *2 / 0.03 =0.24A.

3. Find the separation distance between two plates if the capacitance between them is $1.0 * 2^{-12}$C and the enclosed area is 5.0 $cm^{5.}$ Take $\varepsilon_0 = 6.85 * 2^{-12}Fm^{-1}$. { $d = 1.425 * 2^{-4}$ m}

   Solution

   $C = \varepsilon_0 A/d$

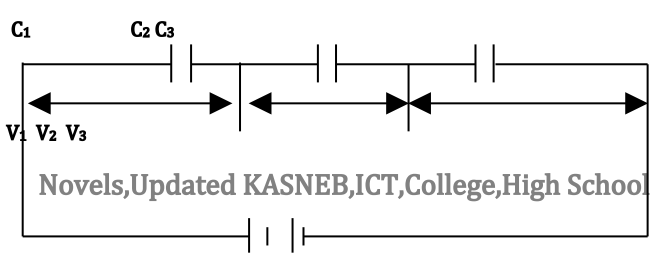
   $d = 6.85 * 2^{-12} * 5.0 * 2^{-4} / 1.0 * 2^{-12}$

   $= 1.425 * 2^{-4}$ m

4.8: Arrangement of capacitors

a) Series arrangement

Consider three capacitors; C_1, C_2 and C_3 arranged as shown below:



V

Recall V = V1 + V2 + V3 and Q = CV

When capacitors are connected in series, the charged stored in them is the same and equals the charge in the circuit.

i.e. $Q = Q_1 = Q_2 = Q_3$

Therefore $V_1 = Q/C_1$, $V_2 = Q/C_2$, and $V_3 = Q/C_3$

$V = Q/C_1 + Q/C_2 + Q/C_3$

Dividing through by Q, we obtain $V/Q = 1/C_1 + 1/C_2 + 1/C_3$

Since $V/Q = 1/C$

$$\mathbf{1/C = 1/C_1 + 1/C_2 + 1/C_3}$$

Where C is the combined capacitance.

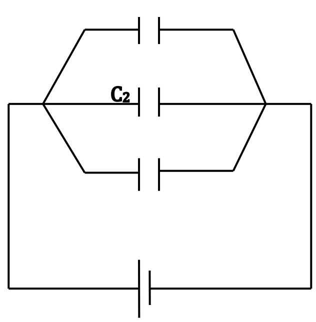
In a special case of two capacitors in series, the effective/combined capacitance**,**

$$\mathbf{C = C_1C_2/(C_1 + C_2).}$$

b) Capacitors in parallel

When capacitors are arranged in parallel, the potential drop across each of them is the same.

C_1



V **C_3**

$Q_1 = C_1V$, $Q_2 = C_2V$, $Q_3 = C_3V$

The total charge, $Q = Q_1 + Q_2 + Q_3$

$Q = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

Dividing through by V, we obtain $Q/V = C_1 + C_2 + C_3$

Since $C = Q/V$,

$$\mathbf{C = C_1 + C_2 + C_3}$$

Hence the combined capacitance for capacitors in parallel is the sum of their capacitance.

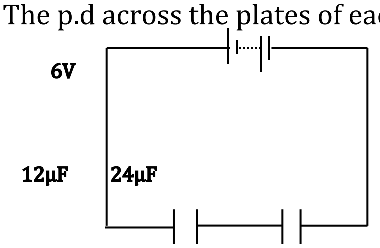
Example 4.2

1. In the circuit below, calculate:

a) The effective capacitance of the capacitors

b) The charge on each capacitor

c) The p.d across the plates of each capacitor



Solution

a) C = 12 * 24 / 12 + 24 =8µF

b) $Q_1 = Q_2$ = CV = 8 * 6 = 48µC

c) V_1 = 48/12 = 4V, V_2 = 48/24 = 2V

2. The figure below shows an arrangement of capacitors connected to a 2V d.c supply.

Determine: a) The combined capacitance of the arrangement

b) The total charge in the circuit

(ans. 0.7778µF,3.778µC)

a) C_{BD} =3*3/3+3 = 1.5µF

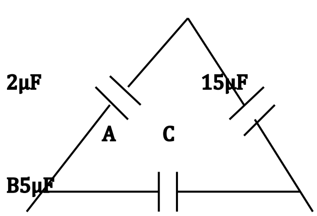
C_{AE} = 2+1.5 = 3.5µF

C = 3.5*1/3.5+1 = 0.7778µF

b) Q = CV = 0.7778*2 = 3.778µC.

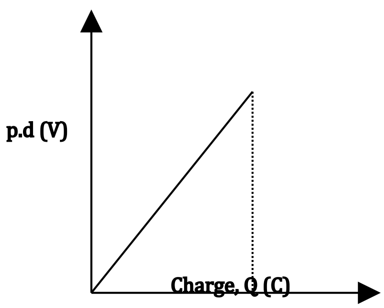
Assignment 1.3

The figure below shows part of a circuit connecting 3 capacitors. Determine the effective capacitance across AC.



4.9: Energy stored by a capacitor

During charging, the addition of electrons to the negatively charged plate involves doing work against the repulsive force. Also the removal of electrons from the positively charged plate involves doing some work against the attractive force. This work done is stored in the capacitor in the form of electrical potential energy. This energy may be converted to heat, light or other forms. A graph of p.d, V against charge, Q is a straight line through the origin whose gradient gives the capacitance of the capacitor.



The area under this graph is equal to the work done or energy stored in the capacitor.

i.e. $E = ½ QV$ but $Q = CV$

Hence **$E = ½ CV^2 = Q^2 /2C$**

Example 1.3

1. The figure below shows two capacitors connected to a 12V supply

Determine: a) the effective capacitance of the circuit

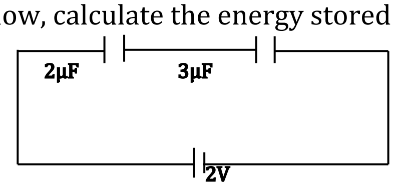
b) Charge on each capacitor

c) Energy stored in the combination

{ans. 18μF, 72μC, 5.46 * 2^{-3}J}

a) 12+6 = 18μF b) Q1= 12*12 =144μC c) E= ½ CV^2 =1/2 *18*2^{-6}*122 =5.46*2^{-3}J

2. In the figure below, calculate the energy stored in the combined capacitor.



{ ans. 5.4*2^{-6})

C = 2*3 /2+3 =1.2μF

E = ½ *1.2 *2-6 *2^2 = 5.4 * 2^{-6} J

45.: Application of capacitors

a) **Rectification (smoothing circuits)**

In the conversion of alternating current to direct current using diodes, a capacitor is used to maintain a high d.c. voltage. This is called smoothing or rectification.

b) **Reduction of sparking in the induction coil**

A capacitor is included in the primary circuit of the induction coil to reduce sparking.

c) **In tuning circuits**

A variable capacitor is connected in parallel to an inductor in the tuning circuit of a radio receiver. When the capacitance of the variable capacitor is varied , the electrical oscillations between the capacitor and the inductor changes. If the frequency of oscillations is equal to the frequency of the radio signal at the aerial of the radio, that signal is received.

d) **In delay circuits**

Capacitors are used in delay circuits designed to give intermittent flow of current in car indicators.

e) **In camera flash**

A capacitor in the flash circuit of a camera is charged by the cell in the circuit. When in use, the capacitor discharges instantly to flash.

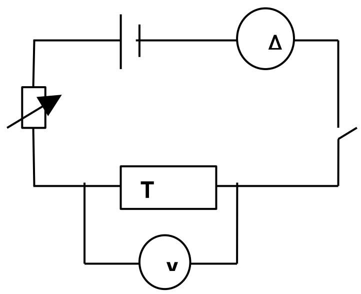
5. CURRENT ELECTRICITY II

5.1: How to use an ammeter and voltmeter

- Connect the positive terminal of the ammeter/ voltmeter to the positive terminal of the battery.
- Ensure that the pointer is initially at zero i.e. there is no zero error. If there is a zero error, correct it before using the instrument.
- Select an appropriate scale to use.
- Avoid parallax error taking readings i.e. view the scale normally.

5.2: Ohm's law

This law relates the current flowing through a conductor and the voltage drop across that section of the conductor. The law states: **the current flowing through a conductor is directly proportional to the potential difference across its ends provided temperature and other physical factors are kept constant**. The following set up can be used to investigate Ohm's law:

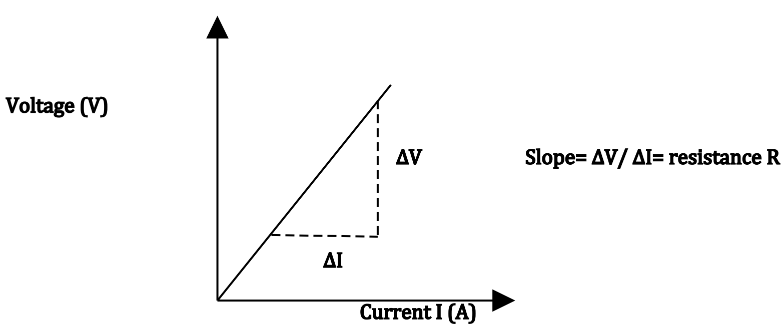


- Close the switch and adjust the current flowing through the conductor T using the rheostat to the least possible value. Record the corresponding voltmeter reading.
- Increase the current in steps recording the corresponding voltmeter readings. Record your values in the table below:

Current I (A)					
Voltage V (V)					

- Plot a graph of voltage against current. Hence determine the slope of the graph.

A graph of voltage against current is a straight line through the origin. Hence voltage drop across the conductor is directly proportional to the current through it;



$V \alpha I$

V/I = constant

The constant is known as resistance R of the conductor T under investigation.

Thus, V/I= R

Or V= IR.

Hence the slope of a voltage—current graph is equal to the resistance R of the conductor T. electrical resistance can be defined as the opposition offered by a conductor to the flow of electric current. It is measured using an ohmmeter.

The SI Unit of electrical resistance is the ohm (Ω). Other units include kilo-ohm (kΩ) and mega-ohm (MΩ);

1Ω= 2^{-3}kΩ

1Ω= 2^{-6}MΩ

Materials which obey Ohm's law are said to be ohmic materials while those which do not obey the law are said to be non-ohmic materials. The graph of voltage against current for non-ohmic materials is a curve or may be a straight line but does not pass through the origin.

The inverse on resistance is called conductance;

Conductance= 1/ resistance R.

Example 5.3

1. Calculate the current flowing through a 8Ω device when it is connected to a 12V supply.

   I= V/R

   I= 12V/8Ω =1.5A

5.5.1: Factors affecting the resistance of a conductor

There are three main factors that affect the resistance of a conductor:

a) **Temperature**

Increase in temperature enhances the vibration of the atoms and thus higher resistance to the flow of current.

b) **Length of the conductor L**

The resistance of a uniform conductor increases with increase in length.

c) **Cross section area A**

A conductor having a wider cross section area has more free electrons per unit length compared to a thin one. Hence a thicker material has a better conductivity than a thinner one. Generally, resistance varies inversely as the cross section area of the material.

Therefore, at a constant temperature resistance varies directly as the length and inversely as the cross section area of the conductor;

RαL/A

R= (A constant * L/A)

Or simply, AR/L= constant

The constant is called the resistivity of the material;

Resistivity ϱ= (cross section area A * resistance R) / length L.

Resistivity is measured in ohm-metre (Ωm).

Example 5.4

1. A wire of resistance 3.5Ω has a length of 0.5m and cross section area 6.2 * $2^{-8}m^{5.}$ Determine its resistivity.

   Resistivity ϱ= AR/L = ($6.2*2^{-8}m^2*3.5Ω$)/0.5m

   = $5.74*2^{-7}$Ωm

2. Two conductors A and B are such that the cross section area of A is twice that of B and the length of B is twice that of A. If the two are made from the same material, determine the ratio of the resistance of A to that of B.

   R=ϱL/A

   Therefore, $R_A = ϱ_A L_A/A_A$

   And $R_B = ϱ_B L_B/A_B$

   Where $L_B=2L_A$

   $A_B= 1/2A_A$

   And $ϱ_A= ϱ_B$

   Hence $R_A=ϱ_A L_A/A_A$ and

   $R_B= 2ϱ_A L_A/0.5A_A = 4ϱ_A L_A/A_A$

   Thus $R_A/R_B = \frac{ϱ_A L_A/A_A}{4ϱ_A L_A/A_A} = 1/4$

   $R_A: R_B$= 1:4

5.11: Resistors

A resistor is a specially designed conductor that offers a particular resistance to the flow of electric current. There are three main groups of resistors:

a) Fixed resistors- offer fixed values of resistance. They have colour bands around them.

b) Variable resistors- offer varying resistance e.g rheostat and potentiometer.

c) Non-linear resistors- the current flowing through these resistors does not change linearly with the voltage applied. Examples include a thermistor and light-dependent resistor (LDR).

5.11.1: Measurement of resistance

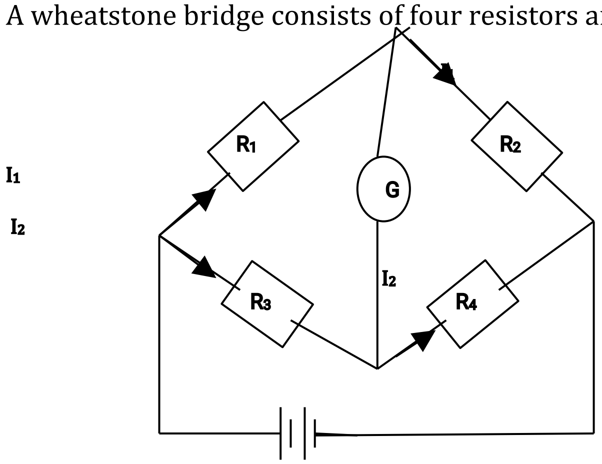
Three methods may be used:

a) Voltmeter- ammeter method

In this method, the current flowing through the material and voltage across its ends are measured and a graph of voltage against current plotted. The slope of the graph gives the resistance offered by the material.

b) The wheatstone bridge method

A wheatstone bridge consists of four resistors and a galvanometer connected as shown below:



The values of three out of the four resistors must be known. The value of one of the resistors is adjusted to a point that the galvanometer does not deflect. At this point, the voltage drop across R_1 is equal to that across R_3. Similarly, the voltage drop across R_2 is equal to that across R_1. Note that the current flowing through R_1 is equal that through R_5. Also, the current through R_3 is the same to that through R_1.

Therefore, $I_1 R_1 = I_2 R_3$………………………… i

$I_1 R_2 = I_2 R_4$………………………… ii

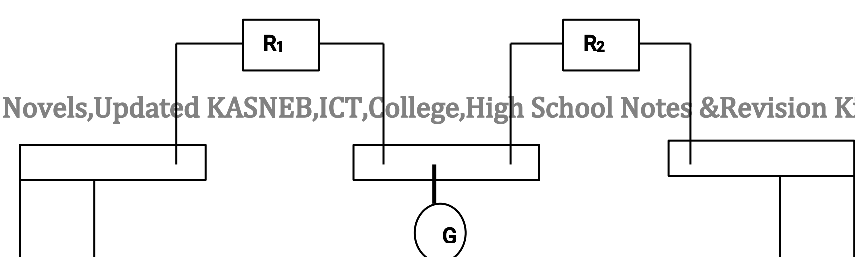
Dividing equation (i) by (ii), we get;

$R_1/R_2 = R_3/R_4$

This method is more accurate compared to the voltmeter- ammeter method since the voltmeter has some resistance against the flow of current and thus takes up some voltage.

c) The metre bridge method

This method relies on the fact that resistance is directly proportional to the length of the conductor.



L_1 K L_2

The values of R_1 and R_2 must be known. Suppose at point K the galvanometer does not deflect, then the voltage drop across R_1 equal the voltage drop across the section L_1. Similarly, the voltage drop across R_2 equals the voltage drop across the section L_5. If the current through R_1 and R_2 is I_1 and that through the section L_1 and L_2 is I_2, then;

$I_1R_1= I_2L_1$ ………………………... i

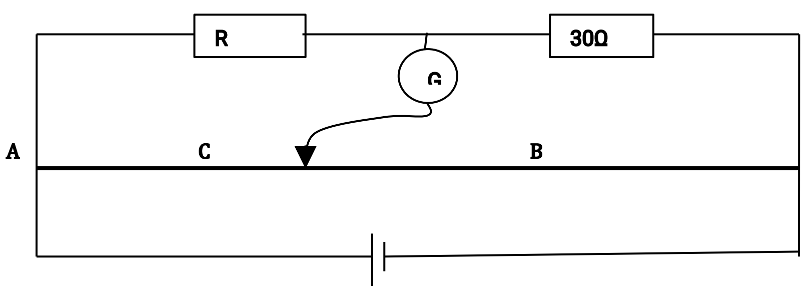
$I_1R_2=I_2L_2$ ………………………… ii

Dividing equation (i) by (ii), we get;

$R_1/R_2= L_1/L_2$

Example 5.5

1. In an experiment to determine the resistance of a nichrome wire using the metre bridge, the balance point was found to be at the 40cm mark. Given that the value of the resistor to the right is 30Ω, calculate the value of the unknown resistor R.



$L_{AC}/L_{CB} = R/30\Omega$

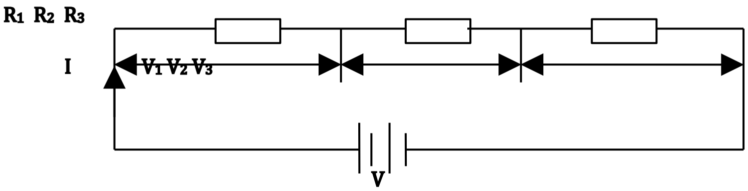
$40cm/60cm = R/30\Omega$

$R= (30*40)/60 = 20\Omega$

5.11.2: Resistor networks

a) Series network

When resistors are arranged in series the same current pass through each one of them. Consider three resistors connected as shown below:



From Ohm's law, V= IR.

The voltage drop across R_1; $V_1= IR_1$

The voltage drop across R_2; $V_2 =IR_2$

The voltage drop across R_3; $V_3=IR_3$

And the total circuit voltage $V= V_1+V_2+V_3$.

Thus $V= IR_1+IR_2+IR_3=I(R_1+R_2+R_3)$

$V/I =(R_1+R_2+R_3)$

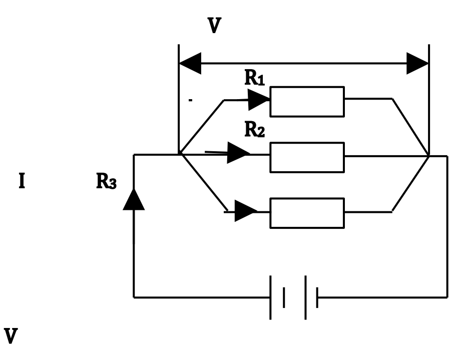
But $V/I = R$

Thus the combined circuit resistance $R=R_1+R_2+R_3$.

Generally, the effective resistance of resistors arranged in series is equal to the sum of the individual resistances.

a) Parallel network

When resistors are connected in parallel, the same voltage is dropped across them. Consider three resistors connected as shown below:



Suppose the current flowing through R_1 is I_1, through R_2 is I_2 and through R_3 is I_3 then:

The voltage drop across R_1; $V_1=I_1R_1$

The voltage drop across R_2; $V_2=I_2R_2$

The voltage drop across R_3; $V_3=I_3R_3$

But $V_1= V_2= V_3= V$ and $I=I_1+I_2+I_3$

Therefore, $I=V/R_1 + V/R_2 + V/R_3$

$I/V = (1/R_1 + 1/R_2 + 1/R_3)$

But $I/V= 1/R$.

Hence $1/R= 1/R_1 + 1/R_2 + 1/R_3$

R is the combined circuit resistance.

Special case of two resistors in parallel

It follows that $1/R= 1/R_1+1/R_2$

$1/R= (R_1+R_2)/R_1R_2$

Hence the effective resistance $R= R_1R_2/ (R_1+R_2)$.

Generally for n resistors arranged in parallel, the effective resistance of the arrangement is given by; $1/R=1/R_1+1/R_2+\ldots\ldots\ldots\ldots+1/R_n$

NOTE: when a circuit comprise of both series and parallel connections, the arrangement is systematically reduced to a single resistor.

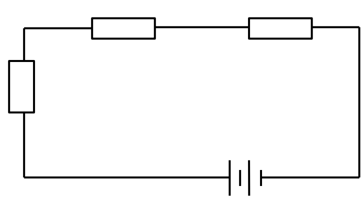
Example 5.6

1. The figure below shows 3 resistors.

5Ω3Ω

8Ω

12V



Calculate:

a) The effective resistance of the circuit.

R= (8+5+3)Ω = 3Ω

b) The total current in the circuit.

I=V/R = 12V/3Ω = 0.75A

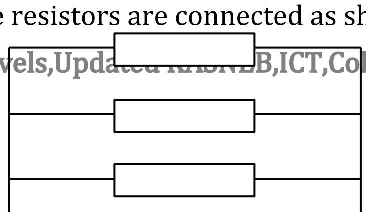
c) The voltage drop across each resistor.

$V_{8\Omega}$= 0.75*8 = 5.0V

$V_{5\Omega}$= 0.75*5 =3.75V

$V_{3\Omega}$=0.75*3 =5.25V

2. Three resistors are connected as shown below:



5Ω

3Ω

6Ω

12V

Calculate:

a) The total resistance of the circuit.

1/R= 1/5+1/3+1/6

1/R= (6+2+5)/30 =21/30

R= 30/21 = 1.4286Ω

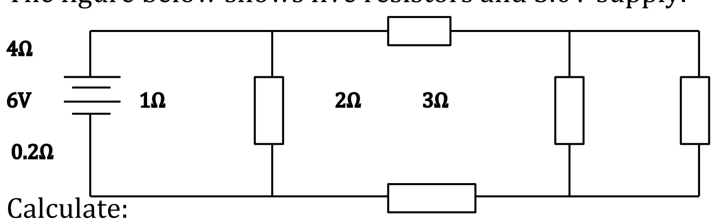
b) The current through each resistor.

$I_{5\Omega}$=12V/5Ω=5.4A

$I_{3\Omega}$=12V/3Ω=1.0A

$I_{6\Omega}$=12V/6Ω=5.0A

3. The figure below shows five resistors and 5.0V supply.



Calculate:

a) The effective resistance of the circuit.

$R_{2,3\Omega}$=(2*3)/(2+3) = 1.2Ω

$R_{4,1.2,0.2\Omega}$=4+1.2+0.2 =5.4Ω

R= $R_{1,5.4\Omega}$=(1*5.4)/(1+5.4) = 0.8438Ω

b) The total circuit current.

I=V/R =6V/0.8438Ω =3.127A

5.11.3: Internal resistance r

When a cell supplies current in a circuit, the potential difference between its terminals is observed to be lower than its electromotive force (emf). This difference is due to the internal resistance of the cell. Some work must be done to overcome this resistance and so the drop in the emf of the cell is responsible for this. The difference is referred to as the **lost volt** and is given by **Ir.**

i.e. lost volts= emf-terminal voltage

Or simply **emf= terminal voltage + lost volts**

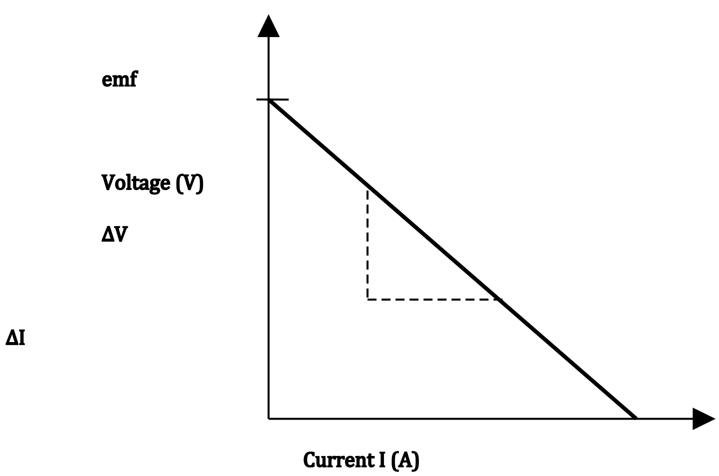
The mathematical equation connecting emf, circuit current, external resistance and internal resistance of the cell is given by:

E= IR + Ir= I(R+r).

Internal resistance of a cell can be obtained experimentally. In such an experiment, the following data was obtained:

Current I(A)	0.1	0.2	0.3	0.4	0.6	0.8
Voltage V(V)	1.43	1.30	1.4	1.09	0.82	0.58

When a graph of Voltage V against current I is plotted, the graph will appear as shown below:



The slope of the graph= **-r (-internal resistance)** while the y-intercept= emf of the cell.

TOPIC 6.: HEATING EFFECT OF ELECTRIC CUURENT

6.1: Introduction

When current flows through a conductor, heat energy is generated in the conductor. The heating effect of an electric current depends on three factors:

- The resistance, R of the conductor. A higher resistance produces more heat.
- The time, t for which current flows. The longer the time the larger the amount of heat produced
- The amount of current, I. the higher the current the larger the amount of heat generated.

Hence the heating effect produced by an electric current, I through a conductor of resistance, R for a time, t is given by $H = I^2Rt$. This equation is called the Joule's equation of electrical heating.

6.2: Electrical energy and power

The work done in pushing a charge round an electrical circuit is given by w.d = VIt

So that power, P = w.d /t = VI

The electrical power consumed by an electrical appliance is given by $P = VI = I^2R = V^2/R$

Example 6.1

1. An electrical bulb is labeled 20W, 240V. Calculate:

a) The current through the filament when the bulb works normally

b) The resistance of the filament used in the bulb.

{ **ans. 0.437A, 575.04Ω**}

Solution

a) I = P/V = 20/240 = 0.437A

b) $R = P/I^2 = 20/\ 0.437^2 = 575.04\Omega$ or $R = V^2/P = 240^2/20 = 576\Omega$

2. Find the energy dissipated in 5 minutes by an electric bulb with a filament of resistance of 500Ω connected to a 240V supply. { **ans. 34,560J**}

   Solution

   $E = Pt = V2/R * t = (240^2 * 5 * 60)/500 = 34{,}560J$

3. A 5.5 kW immersion heater is used to heat water. Calculate:

   a) The operating voltage of the heater if its resistance is 24Ω

   b) The electrical energy converted to heat energy in 2 hours.

   {**ans. 241.9488V, $1.8*2^7$J**}

Solution

a) $P=VI=I^2R$

$I = (2500/24)^{1/2} = 5.2062A$

$V=IR= 5.2062 * 24 = 241.9488V$

b) $E = VIt = Pt = 2500*2*60*60 = 1.8 * 2^7J$

OR $E= VIt = 241.9488 * 5.2062 * 2 * 60 * 60 = 1.8 * 2^7J$

4. An electric bulb is labeled 20W, 240V. Calculate:

a) The current through the filament

b) The resistance of the filament used in the bulb.

{**ans. 0.437A, 575.95Ω**}

Solution

a) $P = VI \quad I = P/V = 20/240 = 0.437A$

b) From Ohm's law, $V = IR \quad R=V/I = 240/0.437 = 575.95Ω$

6.3: Applications of heating effect of electric current

Most household electrical appliances convert electrical energy into heat by this means. These include filament lamps, electric heater, electric iron, electric kettle, etc.

In lighting appliances

a) Filament lamps- it is made of a tungsten wire enclosed in a glass bulb from which air has been removed. This is because air would oxidize the filament. The filament is heated up to a high temperature and becomes white hot. Tungsten is used due its high melting point; 3400^0C. The bulb is filled with an inactive gas e.g. argon or nitrogen at low pressure which reduces evaporation of the tungsten wire. However, one disadvantage of the inert gas is that it causes convection currents which cool the filament. This problem is minimized by coiling the wire so that it occupies a smaller area which reduces heat loss through convection.

b) Fluorescent lamps- these lamps are more efficient compared to filament lamps and last much longer. They have mercury vapour in the glass tube which emits ultraviolet radiation when switched on. This radiation causes the powder in the tube to glow (fluoresce) i.e. emits visible light. Different powders produce different colours. Note that fluorescent lamps are expensive to install but their running cost is much less.

In electrical heating

c) Electric cookers- electric cookers turn red hot and the heat energy produced is absorbed by the cooking pot through conduction.

d) Electric heaters- radiant heaters turn red at about 900^0C and the radiation emitted is directed into the room by polished reflectors.

e) Electric kettles- the heating element is placed at the bottom of the kettle so that the liquid being heated covers it. The heat is then absorbed by water and distributed throughout the whole liquid by convection.

f) Electric irons- when current flows through the heating element, the heat energy developed is conducted to the heavy metal base raising its temperature. This energy is then used to press clothes. The temperature of the electric iron can be controlled using a thermostat (a bimetallic strip).

TOPIC 7.: QUANTITY OF HEAT

7.1: Introduction

When heat is transferred from one body to another, the body which loses heat has its temperature lowered while that which gains heat has its temperature raised.

7.2: Terms used

Heat capacity, C.

This is the quantity of heat energy required to raise the temperature of a given mass of substance by one Kelvin.

i.e. heat capacity, **$C = Q\ (J)/\Delta\theta\ (K)$**

Hence the SI Unit of heat capacity is joule per Kelvin (JK^{-1}).

Specific heat capacity, c

This is the quantity of heat energy required to raise the temperature of a unit mass of a substance by one Kelvin. i.e. **$c = Q\ (J)/m\Delta\theta\ (KgK)$**

$Q = mc\Delta\theta$

The SI Unit of specific heat capacity is joules per kilogram per Kelvin ($JKg^{-1}K^{-1}$).

Note that **$c = C/m$**

Therefore heat capacity, C = mass, m * specific heat capacity, c.

The table below shows some substances with their specific heat capacities:

7.5.1:

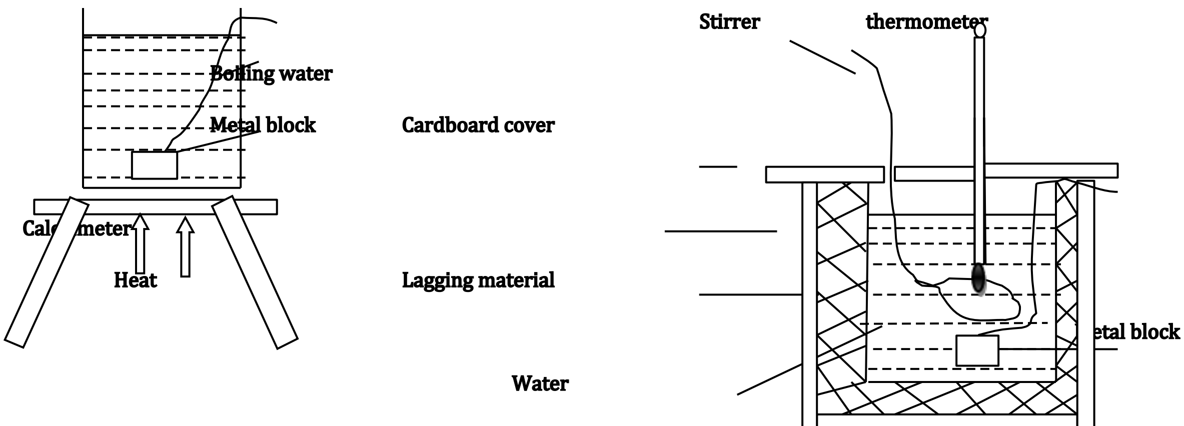
Material	s.h.c ($JKg^{-1}K^{-1}$)
Water	4200
Alcohol	2300
Kerosene	2200
Ice	220
Aluminium	900
Glass	830
Iron	460
Copper	390
Mercury	140
Lead	130

Determination of the specific heat capacity

By the method of mixtures

a) S.h.c of solids

In this method, a known mass of a solid, e.g. a metal block is heated by dipping it in a bath of hot water. After some time, the solid is very fast transferred into cold water in a calorimeter and whose mass is known.



The calorimeter is then covered using a piece of cardboard and stirred continuously. The following measurements are then recorded:

- Mass of the solid metal block, m_s
- Mass of copper calorimeter with the stirrer, m_c
- Mass of the calorimeter and stirrer + water, m_1
- Temperature of the boiling water (initial temperature of the metal block), θ_s
- Temperature of cold water in the calorimeter (initial temperature of calorimeter), θ_w
- Final steady temperature of the mixture, θ

Calculation

Mass of the water in the calorimeter = $m_1 - m_c = m_w$

Temperature change of the hot metal block = $\theta_s - \theta$

Temperature change of the water in the calorimeter and the calorimeter = $\theta - \theta_w$

Assuming there is no heat loss to the surrounding when the metal block is being transferred into the cold water and thereafter;

Amount of heat lost by the metal block = amount of heat gained by calorimeter with stirrer + amount of heat gained by water in the calorimeter.

i.e. $m_s c_s(\theta_s-\theta) = m_c c_c(\theta-\theta_w) + m_w c_w(\theta-\theta_w)$

where c_s – s.h.c. of the metal block

c_c – s.h.c. of the copper calorimeter

c_w – s.h.c. of water.

Hence s.h.c. of the metal block, $c_s=[m_c c_c(\theta-\theta w) + m_w c_w(\theta-\theta_w)] / m_s(\theta_s-\theta)$

b) S.h.c. of a liquid

In this case, a solid of known s.h.c. is used and the water in the calorimeter is replaced with the liquid whose s.h.c. is to be determined. The solid metallic block is first heated in a bath of boiling water and then transferred into the calorimeter containing the liquid. The following measurements are then collected:

- Mass of the metal block, m_s
- Mass of the calorimeter with stirrer, m_c
- Mass of the calorimeter, stirrer and the liquid, m_1
- Initial temperature of the metal block, θ_s
- Initial temperature of the liquid, θ_l
- Final steady temperature of the mixture, θ

If the there is no heat loss to the surrounding, then the quantity of heat lost by the metal block equals the quantity of heat gained by the calorimeter with stirrer and the liquid.

i.e. $m_s c_s(\theta_s - \theta) = [m_c c_c(\theta-\theta_l) + m_l c_l(\theta-\theta_l)]$

Hence $c_l = [m_c c_s(\theta_s - \theta) - m_c c_c(\theta-\theta_l)] / m_l(\theta-\theta_l)$

Alternatively the s.h.c. of a liquid can be obtained by mixing it with another liquid whose specific heat capacity is known and their common temperature determined.

The following precautions must be taken to minimize heat losses to the surroundings:

- Using a highly polished calorimeter
- Heavily lag the calorimeter
- Using a lid of poor thermal conductivity

Example 7.1

1. 70g of a solid initially at 25^0C was carefully dropped into water in a calorimeter at 60^0C. If the final constant temperature of the water and the solid was 54^0C and the mass of water is 500g, determine the specific heat capacity of the solid. Assume the heat absorbed by the calorimeter to be negligible. Take the s.h.c. of water = $4200JKg^{-1}K^{-1.}$

   {**ans. 2, 767.23JKg^{-1}K^{-1}**}

   Solution

   Heat lost = heat gained

   $m_w c_w \Delta\theta_w = m_s c_s \Delta\theta_s$

$0.5Kg * 4200JKg^{-1}K^{-1} * (60-54)\ K = 0.07kg * c_s * (54-25)\ K$

$Cs = 29400J / 5.73KgK = 1O, 767.23\ JKg^{-1}K^{-1}$

2. A student heated 20Kg of water to a temperature of 80^0C. He then added x Kg of water at 15^0C and the final steady temperature of the mixture is 40^0C. Given that the s.h.c. of water is $1.2Jg^{-}1K^{-1}$, determine the value of x. {**ans. 32kg**]

   Solution

Heat lost = heat gained

$20kg * 4200Kg^{-1}K^{-1} * (80-40)\ K = x * 4200JKg^{-1}K^{-1} * (40-15)\ K$

X = 3, 360, 000/25, 000 = 32kg.

3. 0.2kg of iron at 20^0C is dropped into 0.09kg of water at 26^0C inside a calorimeter of mass 0.15kg and s.h.c. $800JKg^{-1}K^{-1}$. Find the final temperature of the water. Take the s.h.c. of iron = $460JKg^{-1}K^{-1}$ and that of water = $4200JKg^{-1}K^{-1}$.

   {**ans. 33.2^{0}C**}

   Solution

Heat lost by iron = heat gained by calorimeter + heat gained by water.

$0.2kg * 460JKg^{-1}K^{-1} * (20-\theta_c)\ K = 0.15kg * 800JKg^{-1}K^{-1} * (\theta_C-26) + 0.09Kg * 4200JKg^{-1}K^{-1} * (\theta_c-26)$

$9200-92\theta_c = 126\theta_c-3120 + 378\theta_c-9828$

$596\theta_c = 22148$

$\theta_c = 22148 / 596 = 33.2^0C$

4. A certain block is heated such that its temperature is raised from 15^0C to 45^0C. calculate the amount of heat absorbed by the metal if its heat capacity is $460JK^{-1}$ {**13, 800J** }

   Solution

$Q = C * \Delta\theta = 460JK^{-1} * (45-15)\ K = 13, 800J.$

5. In an experiment to determine the specific heat capacity of a metal, a 20g of the metal was transferred from boiling water to a lagged copper calorimeter containing cold water. The water was stirred and a final steady temperature was realized. The following data was recorded:

   -initial temperature of cold water and calorimeter =20^0C

   -temperature of boiling water =99^0C

   -final temperature of water, calorimeter and metal =23.7^0C

   -mass of cold water plus calorimeter =130g

   -mass of calorimeter =50g

Take s.h.c. of water=4200JKg-1K-1, s.h.c. of copper=400JKg-1K-1.

Use the data above to determine:

a) The heat gained by the water and calorimeter

$Q = mc\Delta\theta_w + mc\Delta\theta_c = (0.08*4200*3.7) + (0.05*400*3.7)$

$= 2741.2J$

b) The specific heat capacity of the metal

0.1*c*71.3=2741.2

$C=2741.2/0.1*71.3 = 381.46JKg^{-1}K^{-1}$

c) State the possible sources of error in the value of the s.h.c obtained in the above experiment.

- Heat loss as the metal was being transferred from the boiling water to the calorimeter.
- Error when reading the thermometer (parallax error)

6. 3kg of hot water was added to 9kg of cold water at 2^0C and the resulting temperature was 20^0C.ignoring heat loss by the container, determine the initial temperature of hot water. Take s.h.c of water=$4200JKg^{-1}K^{-1}$.

$mc\Delta\theta_h=mc\Delta\theta_c$

$3*(\theta-20) = 9*2$

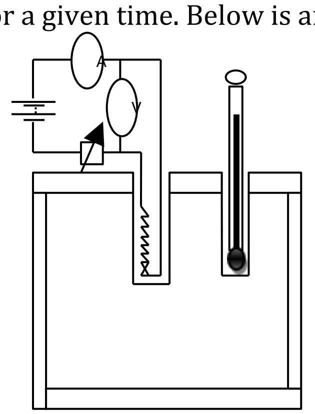
$3\theta=90+60 =150$

$\theta = 150/3 = 50^0C$

Electrical method

a) Specific heat capacity of a solid

In this method, two holes are drilled in the solid to accommodate the heater and thermometer. The solid is heated electrically for a given time. Below is an arrangement that can be used:



In this method, the following data is recorded:

- Mass of the metal (solid)

- Heater voltage, V
- Heater current, I
- Time (duration) of heating, t
- Initial temperature of the solid
- Final temperature of the solid

The electrical energy lost by the heater is given by; E = VIt

Suppose there is no heat loss to the surroundings, then the heat lost by the heater equal heat gained the solid.

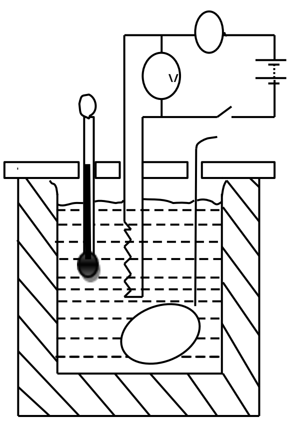
i.e. $VIt = mc\Delta\theta$

Hence $c = VIt/m\Delta\theta$

Note

Heat loss is minimized by lagging the calorimeter as well as oiling the holes.

Specific heat capacity of a liquid



The heat lost by the heater equal the heat gained by the liquid and the calorimeter.

$VIt = mc\Delta\theta_l + mc\Delta\theta_c$

Hence $c_l = (VIt – mc\Delta\theta_c)/m\Delta\theta_l$

Example 7.2

1. An immersion heater rated 120W, 240V is connected to a 240V power supply. How long will it take to heat 1 kg of water from 2^0C to 90^0C? Take s.h.c of water=4200JKg^{-1}K^{-1}.

$t= mc\Delta\theta/VI =mc\Delta\theta/P$

t= (1*4200*80)/120 =2800seconds.

2. A heater rated 180W and a thermometer were inserted in a 0.5kg of water in a copper calorimeter. The following results were recorded:

Temperature,T(^{0}C)	30	36	40	45	49	54	57
Time,t(minutes)	3	4	5	6	7	8	9

a) Plot a graph of temperature against time

b) Use the graph to find:

- The room temperature
- The specific heat capacity of water.

3. A 180W heater is immersed in a copper calorimeter of mass 20g containing 200g of alcohol. When the heater is switched on, after 36 seconds the temperature of the calorimeter and its contents was raised by 120C. Find the specific heat capacity of alcohol. Take the s.h.c of copper=400JKg^{-1}K^{-1}.

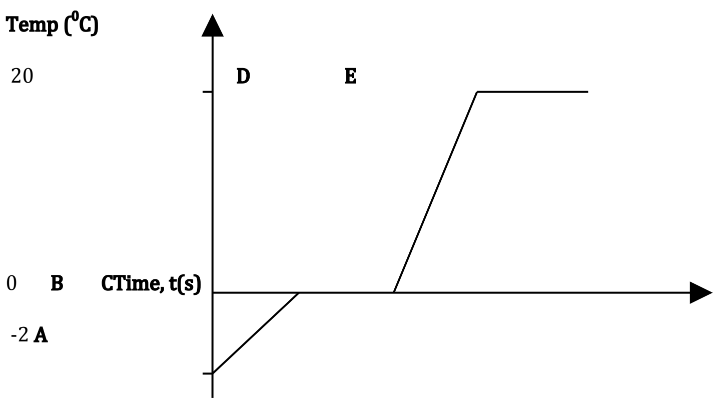
Pt=mcΔθ_a + mcΔθ_c

C_a= (pt-mcΔθ_c)/mΔθ_a= (180*36-0.1*400*12)/0.2*12

= 2500JKg^{-1}K^{-1}

7.3: CHANGE OF STATE

When ice is heated say from -2^{0}C until it boils, it undergoes changes which can be represented by the heating curve below:



Between the points AB, ice absorbs heat energy and its temperature rises. Between BC, the ice absorbs its latent heat of fusion which it uses to melt. This change of state occurs at a constant temperature. Between CD water absorbs heat energy as its temperature rises until boiling point. As the water boils at constant temperature, it absorbs its latent heat of vaporization.

When the vapour condenses to liquid, it gives out its latent heat of vaporization. Similarly, when a liquid freezes to solid, it gives out its latent heat of fusion.

Note:

Latent heat of fusion- it is the quantity of heat needed to convert a given mass of a solid to liquid at constant temperature.

Specific latent heat of fusion- it is the quantity of heat needed to convert a unit mass of a solid to liquid at constant temperature. i.e. **l_f = Q/m**

Therefore Q=ml_f

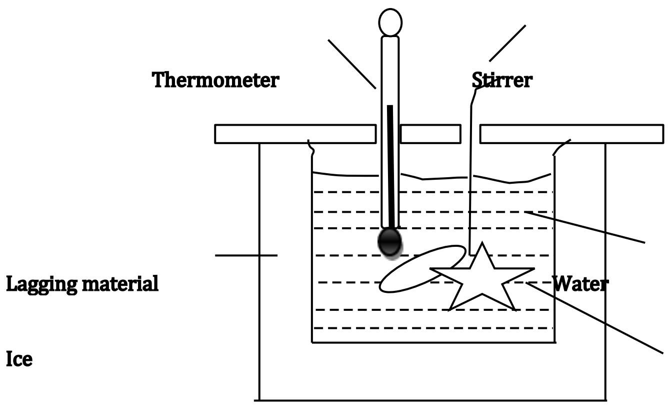
The SI unit of the specific latent heat of fusion is the joule per kilogram (JKg^{-1}). A unit mass of a substance changing from liquid to solid will give out heat energy equivalent to its specific latent heat of fusion.

7.3.1: Determination of specific latent heat of fusion.

There are two methods used:

Mixture method

A piece of dry ice is dropped into a calorimeter containing water slightly above room temperature. Stir the mixture until all the ice has melted. Suppose there is no heat loss to the surroundings, then the heat energy lost by the water and calorimeter equals the heat energy gained by the melting ice



In the above experiment, the following data is recorded for purposes of determining the specific latent heat of fusion:

-Mass of the dry ice

-mass of the water in the calorimeter

-mass of the calorimeter plus stirrer

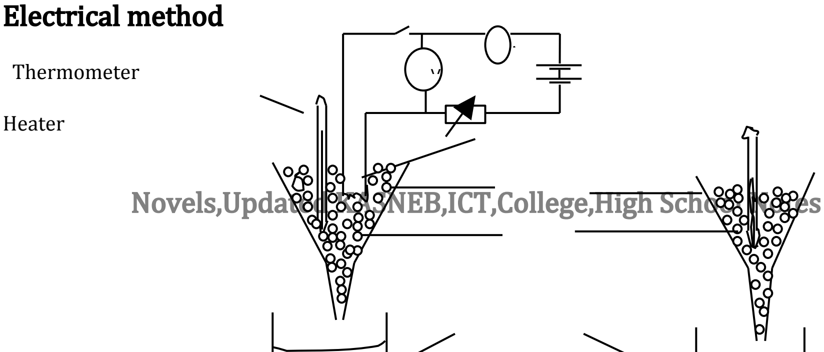
-Temperature change of the water

Hence $mc\Delta\theta_w + mc\Delta\theta_c = ml_f$

$L_f = (mc\Delta\theta_w + mc\Delta\theta_c)/m_i$

NoteDry ice is used due to its low moisture content. This implies that all the heat absorbed by the ice is used to melt the ice and not warming the moisture.

Electrical method



Funnel

Ice

Water

P Q

Equal amounts of crushed ice are put simultaneously in two identical filter funnels. A heater is then immersed in the funnel in set up **P**. Place clean dry beakers below each funnel. Wait until a reasonable amount of water has collected in the beaker **P** then switch off the heater and remove the beakers. Weigh the beakers and their contents.

In the above experiment, the following data is collected:

- Mass of the beaker under **P** before experiment, m_1
- Mass of the beaker under **P** after the experiment, m_2
- Mass of beaker under **Q** before experiment, m_3
- Mass of beaker under **Q** after experiment, m_4
- Heater voltage, V
- Heater current, I
- Duration of heating, t

Calculations

Mass of melted ice in set up **P,** $m_p = m_2 - m_1$

Mass of melted ice in set up **Q**, $m_q = m_4 - m_3$

Set up **Q** is called the control experiment. It helps to determine the mass of ice that melted as a result of the temperature of the room during the experiment. In order to obtain the mass of ice melted by the heater only, it is important to subtract the mass of melted ice in Q from that melted in P;

i.e. $m = m_p - m_q$.

Then, heat energy supplied by the heater = heat energy absorbed by the melting ice.

$VIt = ml_f$

Hence $l_f = VIt/m$

The table below gives some common solids and their specific latent heats of fusion:

Material	**s.l.h of fusion (*2^5) JKg^{-1}**
Copper	1.0
Aluminium	3.9
Water(ice)	3.34

Iron	5.7
Wax	1.8
Naphthalene	1.5
Solder	0.7
Lead	0.026
Mercury	0.013

Example 7.3

1. A block of ice of mass 40g at 0^0C is placed in a calorimeter containing 400g of water at 20^0C. Ignoring heat absorbed by the calorimeter, determine the final temperature of the mixture after all the ice has melted. Take s.h.c.of water= $4200JKg^{-1}K^{-1}$ and the s.l.h. of fusion of ice= 340, $000JKg^{-1}$.

   Heat lost by the hot water= heat gained by melting ice + heat gained by melted ice

   $mc\Delta\theta_h= ml_f + mc\Delta\theta_m$

   $0.4*4200*(20-\theta) = (0.04*340, 000) + (0.04*4200*\theta)$

   $33600-380\theta = 13600 + 38\theta$

   $1848\theta=20000$

   $\Theta=20000/1848 =5.82^0C$

2. 3g of dry ice was added to 20g of water at 26^0C in a beaker of negligible heat capacity. After the ice had all melted, the temperature of water was found to be 12C. Find the specific latent heat of fusion of ice. Take the s.h.c of water =$4200JKg^{-1}K^{-1}$.

   $0.1*4200*(26-11) = (0.03*l_f) + (0.03*4200*11)$

   $6300=0.03l_f + 737.2$

   $L_f=5560.8/0.03 = 3.4755 * 25JKg^{-1}$

3. An aluminium tray of mass 400g containing 300g of water is placed in a refrigerator. After 80minutes, the tray is removed and it is found that 60g of water remains unfrozen at 0^0C. If the initial temperature of the tray and its contents was 20^0C, determine the average amount of heat removed per minute by the refrigerator. Take s.h.c of aluminium = $900JKg^{-1}K^{-1}$, s.h.c of water = 4200JKg-1K-1, s.l.h. of fusion of ice = $3.4*25JKg^{-1}$.

   Heat lost by tray = $mc\Delta\theta = 0.4*900*(20-0)= 7200J$

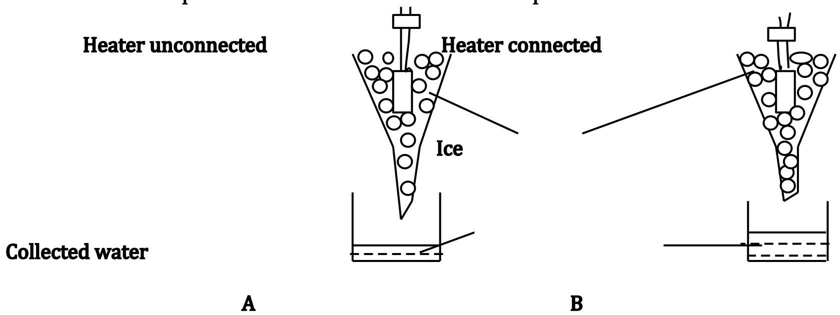
   Heat lost by water = $mc\Delta\theta= 0.3*4200*20 = 25, 200J$

   Latent heat of ice given out = $ml_f = (0.3-0.06)*340, 000 = 999, 600J$

   Total heat energy absorbed by the refrigerator =3600+25200+999600= 114000J

Hence amount of heat removed per minute = 114000J/80min = 1425J/min

4. In an experiment to determine the specific latent heat of fusion of ice, the following set up was used:



In A the heater is unconnected and when the ice is melting steadily, 0.015kg of water is collected in 300s. In B the heater is connected to a power supply rated 50W. When water drips at a steady rate, 0.058kg of water is collected in 300s. Calculate the value for the specific latent heat of fusion of ice.

Q= Pt = ml_f

L_f= (50*300)/(0.058-0.015)

= 348, 833.21JKg^{-1}

Latent heat of vaporization

This is the quantity of heat energy required to convert a given mass of a liquid to gas at constant temperature.

Specific latent heat of vaporization

This is the quantity of heat energy required to convert a unit mass of a liquid to gas at constant temperature.

L_v = Q/m

Therefore, Q= ml_v

The SI unit of specific latent heat of vaporization is the joule per kilogram (JKg^{-1}).

7.3.2: Determination of the specific latent heat of vaporization

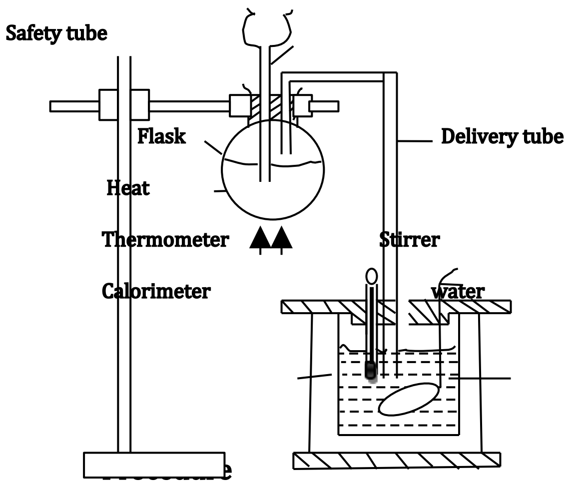
Experiment

Aim: To determine the specific latent heat of vaporization of water using mixture method**.**

Apparatus

- Calorimeter with a stirrer
- Water
- Thermometer
- Flask with a delivery tube
- Heat source

- Weighing machin



1. Set up the apparatus as shown above.
2. Find the mass of the calorimeter when empty and when filled with water to the level shown.
3. Measure and record the initial temperature of water in the calorimeter.
4. Heat the water in the flask until it delivers steam through the delivery tube. Ensure that the free end of the delivery tube is inside water in the calorimeter.
5. Allow steam to bubble into the water while stirring until the temperature of water rises by about 200C above the room temperature.
6. Remove the delivery tube from the calorimeter and record the temperature of the water.
7. Determine the new mass of the calorimeter and its contents. Hence, determine the mass of the condensed steam.

Note

Steam first condenses to water which then cools down, losing heat energy.

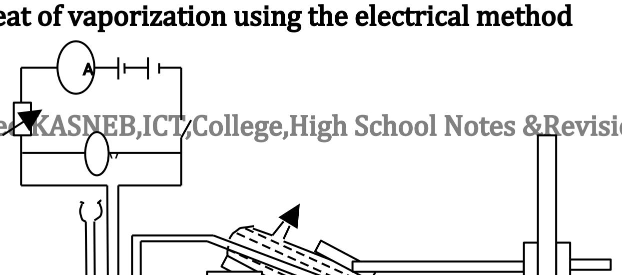
Therefore, heat energy lost by steam and the cooling water equal to the heat energy gained by the water and calorimeter.

$ml_v + mc\Delta\theta_h = mc\Delta\theta_w + mc\Delta\theta_c$

$L_v = (mc\Delta\theta w + mc\Delta\theta_c - mc\Delta\theta_h)/m$

It is important to first cool the water in the calorimeter to a certain value below the room temperature and then pass the steam through it until the temperature rises above the room temperature by the same value. This will help minimize errors due to the heat loss to the surrounding.

Specific latent heat of vaporization using the electrical method



Warm water out

Cold water in

Heater

Condensed water

The heating process is allowed to continue until a steady state where condensed water drips out at a constant rate has been achieved. The mass of water collected after a time, t is measured. The following data is collected in this experiment:

- Heater current, I
- Heater voltage, V
- Mass of empty beaker
- Mass of beaker and collected water
- Time taken to collect the condensed water

Suppose all the heat given by the heater is used to convert water to steam, then:

$VIt = ml_v$

Hence, $l_v = VIt/m$

The table below shows some common liquids and their specific latent heats of vaporization;

Liquid	**s.l.h. of vaporization * 2^5(JKg-1)**
Water	3.6
Alcohol	6.6
Ethanol	6.5
Petrol	5.3
Benzene	1.0
Ether	3.5
Turpentine	5.7

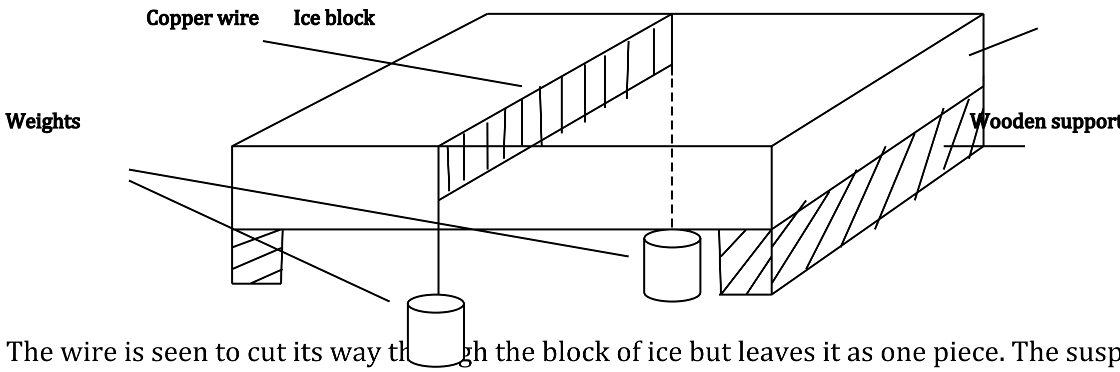
7.4: Boiling and Melting

Boiling and melting points are generally affected by two factors; impurities and pressure.

Melting

1.Effects of pressure on the melting point

Increase in pressure lowers the melting point of a material. This can be illustrated by suspending two weights supported by a copper wire on the surface of an ice block as shown below:



The wire is seen to cut its way through the block of ice but leaves it as one piece. The suspended weights make the copper wire to exert pressure on the ice directly underneath which is made to melt at a temperature below its melting point. As the wire cuts through, the water formed flows over the wire and immediately solidifies since it is no longer under pressure. As the water solidifies, it gives out its latent heat of fusion which is conducted by the copper wire to melt the ice below it. This continues until the copper wire completely cuts through the ice leaving it intact.

Note that copper wire has been used due to its high thermal conductivity. If a poor thermal conductor like cotton string was used, it would not cut through the ice block.

The process by which water refreezes is referred to as **regelation.**

The effects of high pressure on the melting point are applicable in ice skating and joining two pieces of ice blocks together. The weight of the skater acts on the thin blades of the skates exerting high pressure on the ice. The ice underneath thus melts, forming a thin film of water over which the skater slides.

When two ice cubes are pressed hard against each, the high pressure between them lowers the melting point of the ice at the point of contact. When the pressing force is withdrawn, water recondenses and the two cubes are joined together.

5. The presence of impurities lowers the melting point of a material. This is the reason behind spreading salt on roads and paths during winter in cold regions. This will prevent freezing on the roads.

Boiling

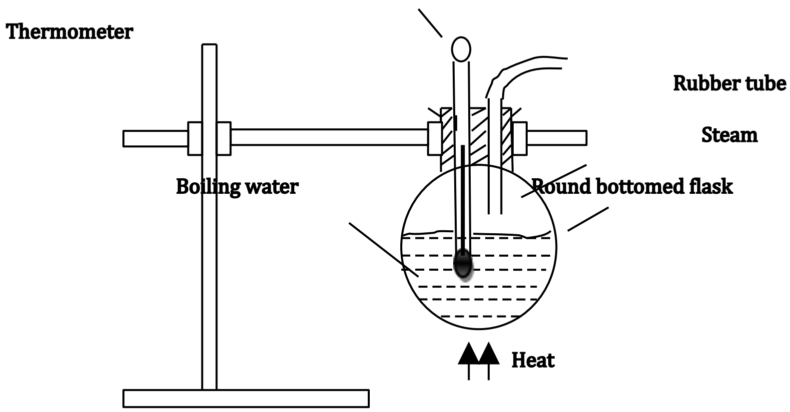
Generally:

-The presence of impurities in a liquid raises the boiling point of the liquid.

-An increase in pressure raises the boiling point of the liquid.

The effects of pressure on boiling point may be illustrated by the set ups below:

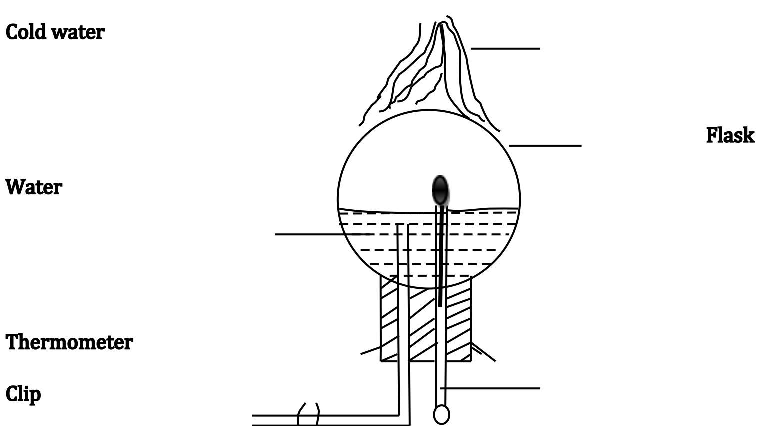
Effects of increased pressure on boiling point



The heating is done until water starts to boil. The temperature at which water boils is noted. When the rubber tube issuing steam is squeezed momentarily, the reading on the thermometer is observed to rise and boiling reduces. Note that closing the tube raises the vapour pressure within the flask. This makes it difficult for the molecules from the surface of the liquid to escape, raising the boiling point of the liquid.

The effect of high pressure on boiling point is applied in a pressure cooker. Here the pressure is raised which raises the boiling point of water hence the food is cooked at a higher temperature.

Effects of reduced pressure on boiling point



Water is first heated to boiling. The flask is then turned upside down and cold water poured over it. It would be observed that when heating stops, boiling also stops. When cold water is poured over the flask, the water inside the flask begins boiling again although its temperature is below the boiling point.

The cold water condenses the steam reducing vapour pressure inside in the flask. Hence a decrease in pressure lowers the boiling point of a liquid.

7.5: Boiling and Evaporation

When a liquid is heated, the molecules close to the surface may gain sufficient kinetic energy to break away from the forces of attraction between the neighboring molecules and escape. This is called **evaporation.** Evaporation takes place at any temperature, even below the boiling point of the liquid.

Factors affecting rate of evaporation

a) Temperature

Increase in temperature of the liquid enhances evaporation. This is why clothes dry faster on a hot day.

b) Surface area

When the surface area is increased, the molecules of the liquid have greater chance of escaping. Hence a wet cloth would dry faster when it is spread out than when it is folded.

c) Humidity

When there is high amount of water vapour in the atmosphere, it becomes difficult for the molecules to escape. This is why clothes take longer to dry on a humid day.

d) Draught/moving wind

Moving air above the surface of the liquid sweeps away the escaping molecules. Thus evaporation is enhanced by the passing air.

DIFFERENCES BETWEEN BOILING AND EVAPORATION

Evaporation	**Boiling**
Occurs at all temperatures	Occurs at a fixed temperature
Occurs at the surface of the liquid	Takes place throughout the liquid
No bubbles are formed	Bubbles are formed in the liquid
Decrease in atmospheric pressure increases the rate of evaporation	Decrease in atmospheric pressure lowers the boiling point of the liquid

Evaporation has a cooling effect which is applied in sweating in human beings and animals, cooling of water in porous pots and the refrigerator.

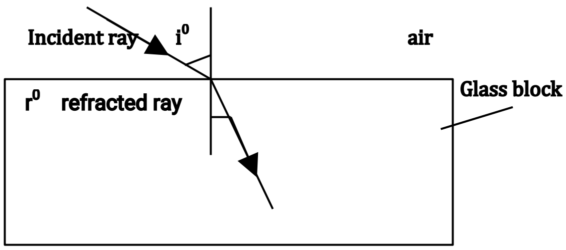
When water evaporates, it absorbs the latent heat from the body causing a cooling effect. Different animals have different ways by which they cool their bodies. For instance, dogs expose their tongues when it is hot while the muzzle of a cow becomes more wet when it is hot. Both these are to increase the rate of evaporation thereby cooling the body.

A porous pot has tiny holes which allow water to seep out slowly. As the water evaporates, it absorbs the latent heat causing a cooling effect.

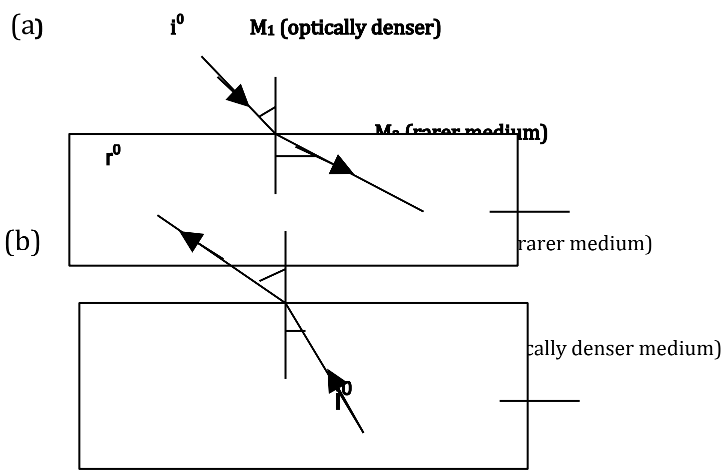
TOPIC 8.: REFRACTION OF LIGHT

8.1: Introduction

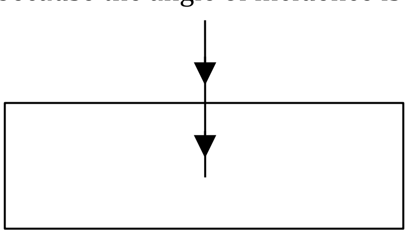
Refraction refers to the bending of light when it passes from one medium into another of different optical density. This is because as light passes through different media its velocity changes. The bending occurs at the boundary or interface of the two media.



The refracted ray may bend away or towards the normal depending on the optical density of the second medium with respect to the first medium. Generally, a ray passing from an optically denser medium into a less optically dense (rarer) medium is bent away from the normal after refraction. If the ray passes from a rarer medium into an optically denser medium then it is bent towards the normal. It is easier to tell which medium is optically denser by simply comparing the angle between the incident ray and the normal and that between the refracted ray and the normal. The medium with a smaller angle (of incidence or refraction) is the optically denser medium.



However, when the ray strikes the interface perpendicularly (normally) it passes undeviated (without bending). This is because the angle of incidence is zero.



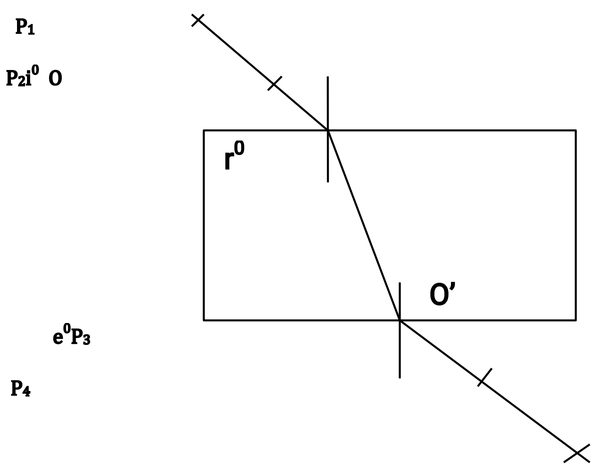
In figure (b) above, only the direction of the light has been reversed leaving the angles the same. However, i now become r while r becomes i. The principle that makes it possible to reverse the direction of light keeping the sizes of the angles the rays make with the normal the same is called the principle of reversibility of light.

The study of refraction of light helps us understand the following common phenomena:

- Why a stick appears bent when part of it is in water.
- Why a coin at the base of a beaker of water appears nearer the surface than it actually is.
- Why the stars twinkle.
- Why the sun can still be seen sometimes before it rises or even after setting.
- Why the summer sky appears blue.
- The formation of the rainbow.

8.2: Refraction in glass

This can be investigated by the following steps:



- Fix a white plain paper on a soft board using drawing pins. Place the glass block with its larger surface on the plain paper and trace its outline.
- Remove the glass block and then draw a normal through point O. Draw a line making an angle say $i=30^0$ with the normal as shown above.
- Replace the glass block onto the outline and stick two pins P_1 and P_2 along the line such that they are upright and about 6cm apart.
- From the opposite side of the block, view the two pins and stick two pins P_3 and P_4 such that the four pins appear on a straight line. Join the positions of P_3 and P_4 using a straight line and produce the line to meet the outline at O'.
- Draw another normal at O' and then join O to O'. Measure angles **r** and **e**.
- Repeat the above steps for other values of $i=40^0$, 50^0 and 60^0. Complete the table below:

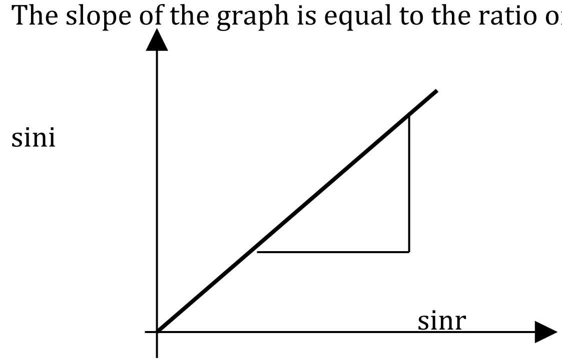
Angle of incidence, i^0	30^0	40^0	50^0	60^0
Angle of incidence, r^0				

e^0				
Sin i				
Sin r				
Sini/Sinr				

- Plot a graph of Sin i against Sin r. determine the slope of your graph.

Observations

- The ratio of Sin i to Sin r is a constant.
- The graph of Sin i against Sin r is a straight line through the origin.
- The slope of the graph is equal to the ratio of Sin i to Sin r in the table.



8.3: The laws of refraction and refractive index

There are two laws of refraction:

1. The incident ray, refracted ray and the normal at the point of incidence all lie in the same plane.

8. **Snell's law**: it states that the ratio of sine of angle of incidence to the sine of angle of refraction is a constant for a given pair of media.

i.e. Sin i/Sin r = a constant.

The constant is referred to as the **refractive index, η** of the second medium with respect to the first medium. The first medium is that medium in which the incident ray is found while the second medium is that medium where the refracted ray is found. It is denoted as $_1\eta_8$.

Hence in 8.2 above, the ratio Sin i/Sin r is the refractive index of glass with respect to the air since the light passed from air into glass block.

However, when light passes from vacuum into another medium, it is referred to as absolute refractive index. Therefore for absolute refractive index, the angle of incidence **i**is found in a vacuum.

i.e. absolute refractive index= sin i(in vacuum)/sin r(in the second medium).

Recall:

$_1\eta_2$=sin i/sin r

By the principle of reversibility of light, r now becomes i and i becomes r i.e. the incident ray is now found in the second medium.

Hence $_2\eta_1$=sin r/sin i

But sin r/sin i=1/(sin i/sin r)=1/$_1\eta_2$

Therefore $_2\eta_1$=1/$_1\eta_8$.

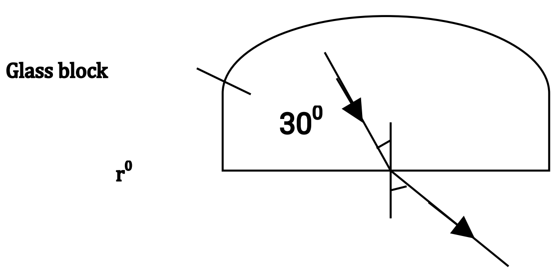
The table below shows some materials and their refractive indices:

Material	Refractive index
Ice	1.31
Crown glass	1.50
Water	1.33
Alcohol	1.36
Kerosene	1.44
Diamond	8.42

Note that the refractive indices given in the above table are with respect to air i.e. when light travels from air into the various media.

Example 8.1

1. In the figure below, calculate the angle of refraction r given that the refractive index of the glass is 1.50.



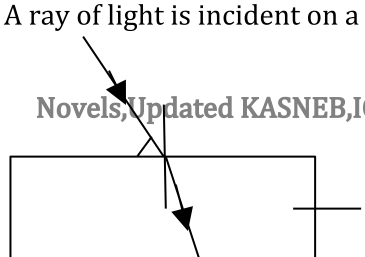
By the principle of reversibility of light;

sin r/sin 30^0= 1.50

sin r=1.50*sin 30^0

r= sin-1(1.50*sin 30^0)= 48.6^0.

2. A ray of light is incident on a flat glass surface as shown below:



55^0

Glass

Given that the refractive index of glass is 1.50, determine the angle of refraction for the ray of light.

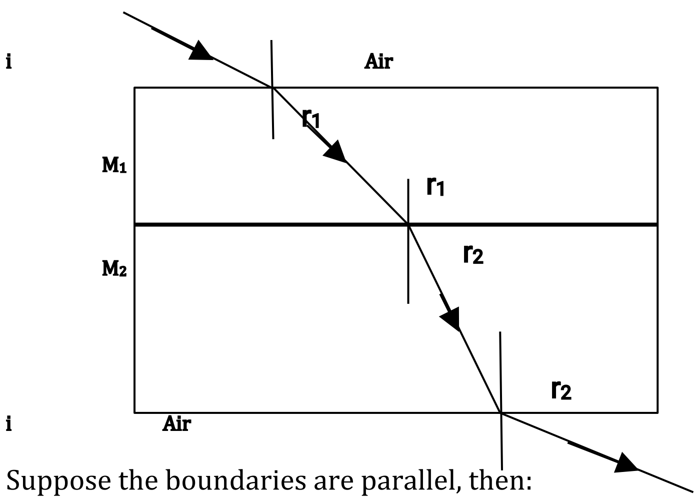
$1.50 = \sin 35^0 / \sin r$

$\sin r = \sin 35^0 / 1.50$

$r = \sin^{-1}(\sin 35^0 / 1.50) = 3.48^0$

8.3.1: Refraction through successive media

Consider a ray of light passing through a series of media as shown below:



Suppose the boundaries are parallel, then:

${}_a\eta_1 = \sin i / \sin r1$……………………………… (i)

${}_1\eta_2 = \sin r1 / \sin r2$ …………………………… (ii)

${}_2\eta_a = \sin r2 / \sin i$ ……………………………… (iii)

By the principle of reversibility of light;

${}_a\eta_2 = \sin i / \sin r2$…………………………… (iv)

Also, multiplying equations (i) and (ii), we get:

${}_a\eta_1 * {}_1\eta_2 = \sin i / \sin r_1 * \sin r_1 / \sin r_2 = \sin i / \sin r_8.$

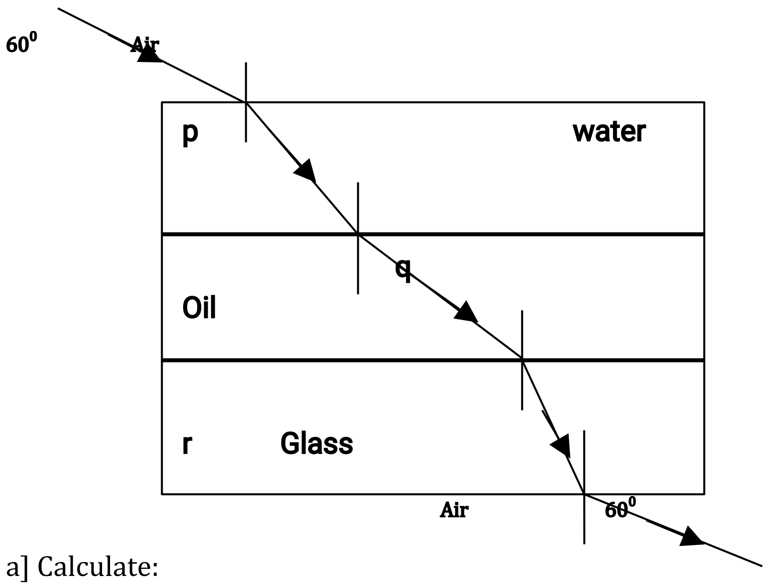
Thus ${}_a\eta_2 = {}_a\eta_1 * {}_1\eta_8.$

Generally, ${}_1\eta_k = {}_1\eta_2 * {}_2\eta_3 * \ldots\ldots\ldots\ldots * {}_{k-1}\eta_k.$

Example 8.2

1. A ray of light from air passes successively through parallel layers of water, oil, glass and then into air again. If the refractive indices of water, oil and glass are 4/3, 6/5 and 3/2 respectively and the angle of incidence in air is 60^0.

a) Draw a diagram to show how the ray passes through the multiple layers.



a] Calculate:

i) The angle of refraction in water.

$4/3 = \sin 60^0/\sin r$

$r = \sin^{-1}(3\sin 60^0/4) = 40.5^0$

ii) The angle of incidence at the oil-glass interface.

${}_o\eta_g = \sin q/\sin r$

By the principle of reversibility of light, ${}_a\eta_g = \sin 60^0/\sin r = 3/2$

$r = \sin^{-1}(2\sin 60^0/3) = 38.27^0$.

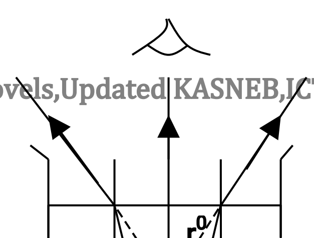
Also, ${}_o\eta_g = {}_o\eta_a * {}_a\eta_g = 5/4$

Therefore, $5/4 = \sin q/\sin 38.27^0$

$q = \sin^{-1}(5\sin 38.270/4) = 48.4^0$

8.4: Refractive index in terms of real and apparent depth

This is on the basis that when an object at the base of a container filled with water is viewed perpendicularly it appears closer to the surface than it actually is. Consider the figure below:



E C r^0 D

Water

A

From the figure, ${}_{w}\eta_{a}$=sin i/sin r.

Therefore, ${}_{a}\eta_{w}$=sin r/sin i.

Since the angles i and r are very small, sin i=tan i and sin r=tan r.

Therefore, by the principle of reversibility of light, ${}_{a}\eta_{w}$=sin r/sin i =tan r/tan i = (CD/BC)/(CD/AC)

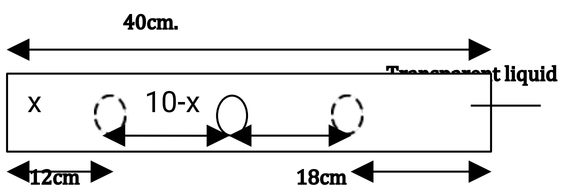
Thus ${}_{a}\eta_{w}$= AC/BC , where AC- real depth and BC- apparent depth.

Hence, refractive index of water= Real depth/Apparent depth.

When a graph of real depth against apparent depth is plotted, the graph obtained is a straight line through the origin and whose gradient is equal to the refractive index of the medium involved.

Example 8.3

1. In a transparent liquid container, an air bubble appears to be 12cm when viewed from one side and 18cm when viewed from the other side. If the length of the tank is 40cm, where exactly is the air bubble?



Refractive index of glass= (12+x)/12 = (18+2-x)/18

x= 20/5 = 4cm.

Therefore, the bubble is 3cm in the liquid from the left-hand side.

8. A microscope is focused on a mark on a horizontal surface. A rectangular glass block 30mm thick is placed on the mark. The microscope is then adjusted 2mm upwards to bring the mark back to focus. Determine the refractive index of the glass.

${}_{a}\eta_{g}$=real depth/apparent depth= 30mm/20mm

=1.50

8.5: Refractive index in terms of velocity of light

Refraction occurs as a result of the different light velocity in different media. Basically, refractive index of any medium is the ratio of the velocity of light in a vacuum or air to the velocity of light in that medium;

η_m= velocity of light in vacuum/velocity of light in the medium .

Note that the velocity of light in a vacuum is $3.0*2^8$m/s.

Generally, ${}_1\eta_2$=velocity of light in medium 1/velocity of light in medium 8.

Example 8.4

1. The velocity of light in glass is $8.0*2^8$m/s. Calculate:

a) The refractive index of glass.

η_g= velocity of light in vacuum/velocity of light in glass= $(3.0*2^8)/(8.0*2^8)$ =1.50

b) The angle of refraction in glass for a ray of light incident at the air-glass interface at an angle of incidence of 40^0.

Sin 40^0/sin r =1.50

r=sin-1(sin40/1.50)= 10.4^0.

8. Calculate the speed of light in diamond of refractive index 8.1.

η_d=velocity of light in vacuum/velocity of light in diamond

8.4= $(3.0*2^8)/V_d$

$V_d=(3.0*2^8)/8.4 =1.25*2^8$m/s.

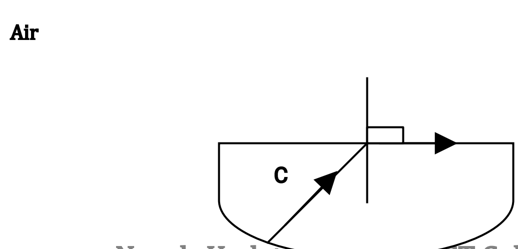
3. The speed of light in medium 1 is $8.0*2^8$m/s and in medium 2 is $1.5*2^8$m/s. Calculate the refractive index of medium 2 with respect to medium 1.

${}_1\eta_2=V_1/V_2$ = $(8.0*2^8$m/s$)/(1.5*2^8$m/s$)$

=1.33

8.6: Total internal reflection, critical angle and refractive index

As the angle of incidence in the denser medium increases the angle of refraction also increases. If this continues until the angle of refraction reaches 90^0, the angle of incidence is called the critical angle C. A critical angle is defined as the angle of incidence in the denser medium for which the angle of refraction is 90^0 in the less dense medium.



By the principle of the reversibility of light,

$_a\eta_g$= sin90^0/sin C =1/sin C.

If the angle of incidence exceeds the critical angle, the light undergoes total internal reflection. This reflection obeys all the laws of reflection.

For total internal reflection to occur, two conditions must be satisfied, namely:

- ✓ Light must pass from an optically denser medium to a less optically dense medium.
- ✓ The angle of incidence in the denser medium must be greater than the critical angle.

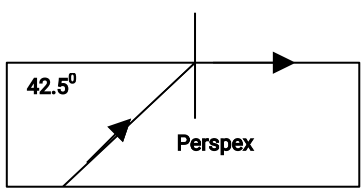
Example 8.5

1. Calculate the critical angle for glass whose refractive index is 1.50.

1.50= 1/sin C.

C = sin-1(1/1.50) =

8. The figure below shows the path of a ray light passing through a rectangular block of Perspex placed in air.



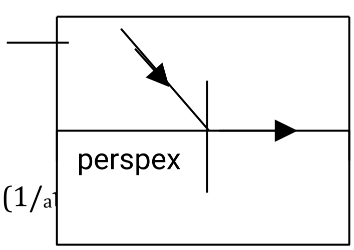
a) Calculate the refractive index of Perspex.

$_a\eta_p$=1/sin48.5^0= 1.48

b) A ray of light now travels from a transparent medium of refractive index 8.4 into the Perspex as shown below:

Transparent material

C



Calculate the critical angle C.

$_p\eta_m$= sin C/sin 90^0 = $_p\eta_a$*$_a\eta_m$=(1/$_a$

=1/8.4 *1.48=1.48/8.4

C= sin-1(1.48sin90^0/8.4) =38.07^0.

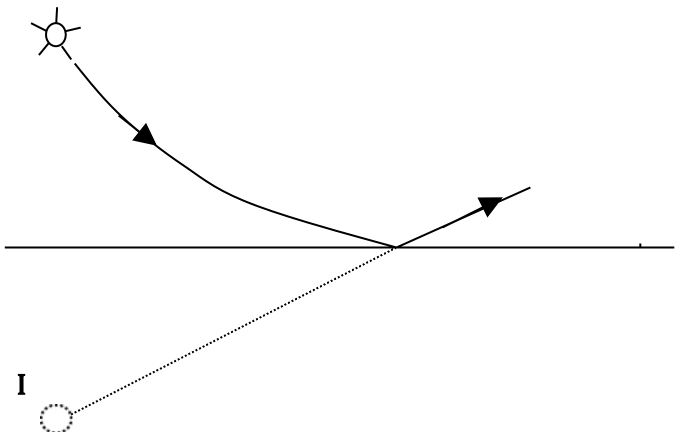
8.8.1: Effects of total internal reflection

- ❖ **Mirage**

On a hot day, the air above the ground is at a higher temperature than the layers above it. Thus the density of air increases with height above the ground. Denser air is optically denser than lighter one. Hence, a ray of light from the sun undergoes continuous refraction at the boundaries between any two layers of air with different temperatures.

In each case, the ray bends away from the normal until the critical angle is achieved. Thereafter, the ray undergoes total internal reflection. An inverted image in the form of a pool of water is observed. This phenomenon is referred to as **mirage**.

Generally, mirage occurs as a result of continuous and progressive refraction at the air boundaries and total internal reflection. Mirage also occurs in cold regions but this time the ray of light curves upwards.



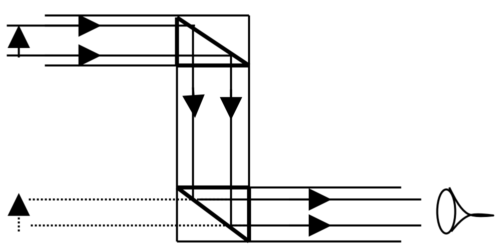
- **Atmospheric refraction**

The sun is sometimes seen before it actually rises or after it has set. This is because the light from the sun is refracted by the atmosphere towards the earth. (**Recall: the earth is spherical**).

8.8.2: Applications of total internal reflection

a) A prism periscope

It makes use of two right-angled isosceles prisms. The light from the object is inverted through 90^0 by the first prism and a further 90^0 by the second prism.



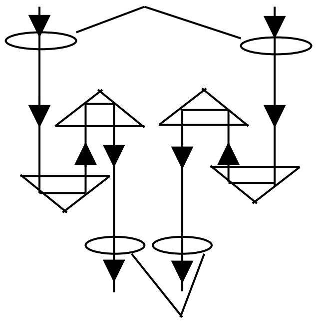
This periscope produces brighter images compared to those of the simple periscope in which a plane is used. The image formed is erect and virtual. A prism periscope has the following advantages over the simple periscope:

- ✓ Forms brighter and clearer images. A simple periscope produces many faint images besides the main image especially if the mirror is thick.
- ✓ Does not absorb the energy of the light. Plane mirrors absorb some light incident on them.
- ✓ Has a tough structure and thus does not easily wear. The painting on the plane mirror can wear out with time.

b) A prism binoculars

This device is used to reduce the distance between the eyepiece and the objective thereby reducing the length of the telescope. It forms an erect image.

Objective lenses

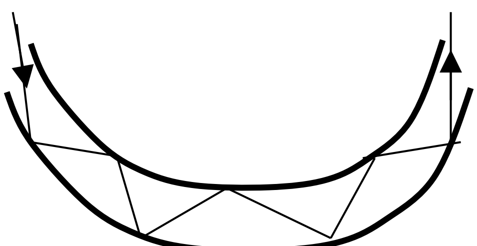


Eyepiece lenses

c) Optical fibre

It is a thin flexible glass rod made up of two parts; the inner part made of glass of higher refractive index and the outer glass coating of lower refractive index. When a ray of light enters the fibre at an angle greater than the critical angle, it undergoes a series of total internal reflection before it finally emerges from the other end. None of the light energy is lost in the process.

Optical fibres are used in medicine for viewing internal body organs (the endoscope) as well as in telecommunication. They are preferred to ordinary cables because they are light and thin and do not cause scattering of the signals.

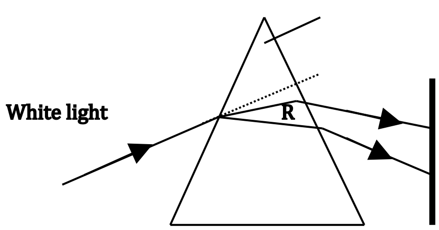


8.7: Dispersion of light

White light from the sun is made up of seven colours. They all travel with the same velocity in vacuum but their velocities vary in other transparent media like glass and water. Hence when a ray of white light travels from a vacuum into a glass prism, it is separated into its component colours ranging from red, orange, yellow, green, blue, indigo to violet. The spreading out of light into its constituent colours by another medium is called **dispersion**. Pure light is called **monochromatic** light while an impure light like white light is referred to as **non-monochromatic or composite** light. Dispersion of light is illustrated by the diagram below:

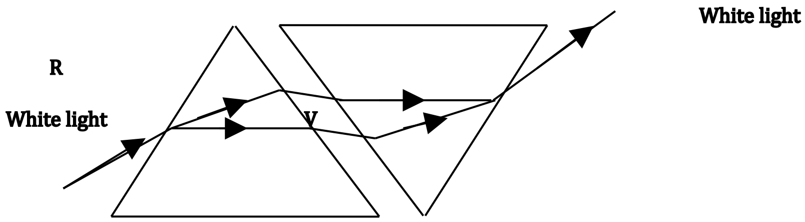
Glass prism

V



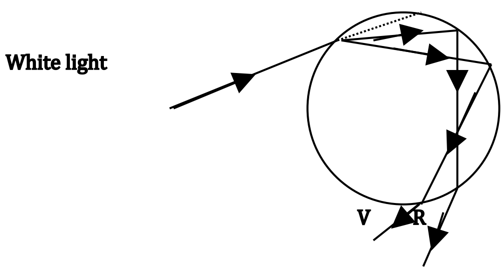
Red is least deviated while violet is the most deviated ray. Hence red light has the greatest velocity and violet the least velocity in glass. The coloured band produced is called a **visible spectrum**. The spectrum produced above is impure. In order to obtain a pure spectrum where each colour is distinct, an achromatic lens is placed between the screen and the prism.

When the seven sevencolours are recombined, a white light is obtained. This can be achieved by using a similar but an inverted prism.



8.8: The rainbow

When a ray of light passes through a water drop, a rainbow is produced. The water disperses the light into its constituent colours. Each colour then undergoes total internal reflection within the drop before it eventually emerges into air again.



TOPIC 9.: WAVES
9.1: Introduction

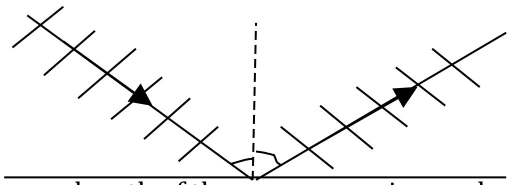
9.2: Properties of waves(form three)

Wave properties refer to the behaviour of waves under certain conditions. They include reflection, refraction, diffraction and interference among others. They can be investigated using a ripple tank which consists of a transparent tray containing water, a lamp for illumination, a white screen underneath and an electric motor (a vibrator). The motor is connected to a straight bar which produces straight waves. If circular waves are required, the bar is raised and a small spherical ball fitted to it to produce circular waves. To view the waves with ease, a stroboscope is used. A stroboscope is a disc having equally spaced slits. It is rotated and its speed controlled such that the waves appear stationary i.e frozen.

9.8.1: Reflection of waves

All waves undergo reflection. It is the bouncing back of waves when they hit an obstacle. All waves undergoing reflection obey the laws of reflection as earlier stated.

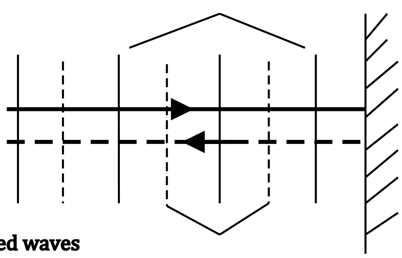
i^0 r^0



Note that the wavelength of the waves remains unchanged. The pattern of the reflected waves depends on the shape of the incident waves and the reflector. Below are some patterns:

a) Plane waves incident on a straight reflector

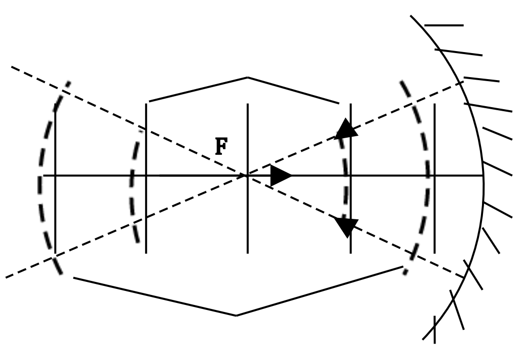
Incident wavefronts



Reflected waves

b) Plane waves incident on a concave reflector

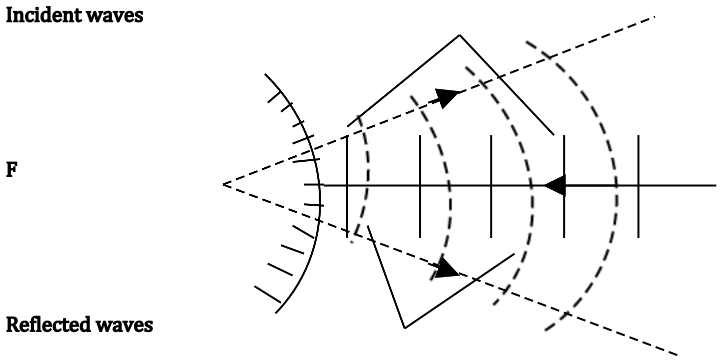
Incident waves



Reflected waves

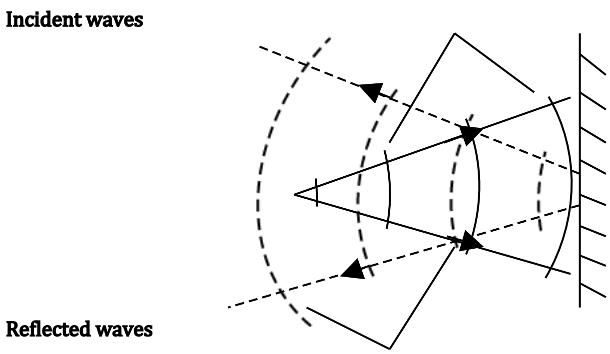
The waves converge at the principal focus F after reflection.

c) Plane waves incident on a convex reflector



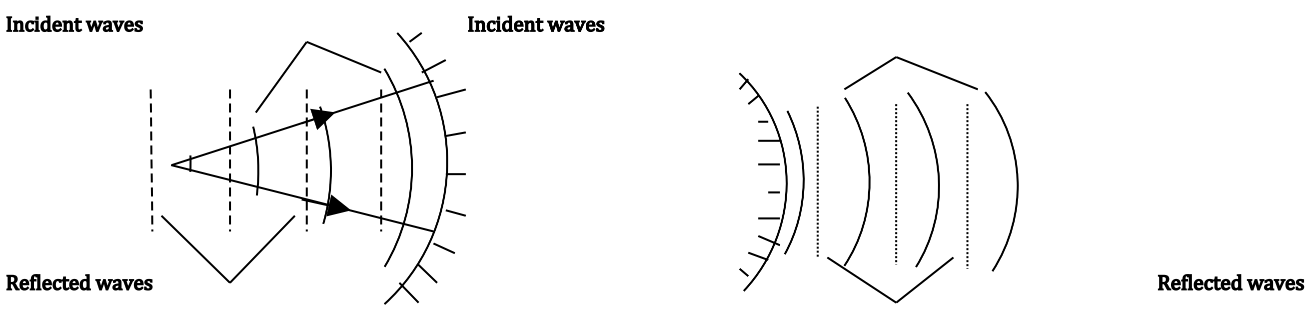
The reflected waves appear to be diverging from a point (principal focus) behind the reflector.

d) Circular waves incident on a straight reflector



The reflected waves diverge away from the reflector.

e) Circular waves incident on a concave/convex reflector

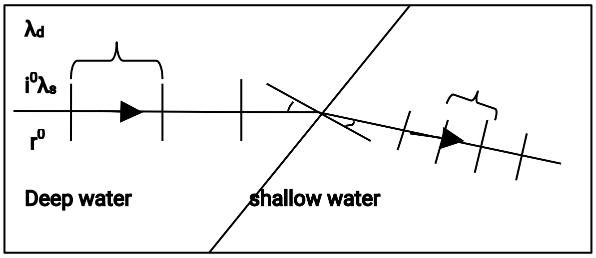


9.8.2: Refraction of waves

This is the bending of waves as they travel from one medium into another. In the process, the speed of the waves changes from one medium to another. In the case of water waves, refraction occurs as the waves move from a region of a certain depth into another region of a different depth i.e. from a shallow region to a deeper region or vice versa. In general, the speed of water waves is greater in a deeper region than in a shallow region. It is important to note that the source of waves remains the same regardless of the depth thereafter. Hence, the frequency of the waves is a constant.

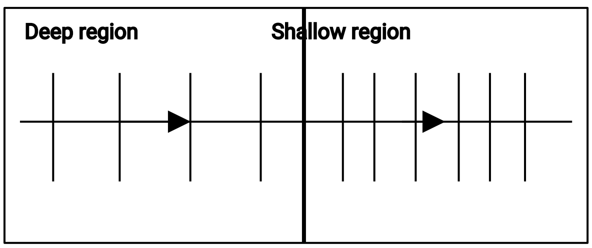
Recal**l: wave speed= frequency f*wavelength λ**.

From the equation, it is clear that when the wave speed increases the wavelength also increases and vice versa. Thus, the wavelength is longer in deeper regions than in shallow regions.



To obtain a shallow region in a ripple tank, a transparent glass block is placed in the tank with one end of its edge parallel to the vibrating bar.

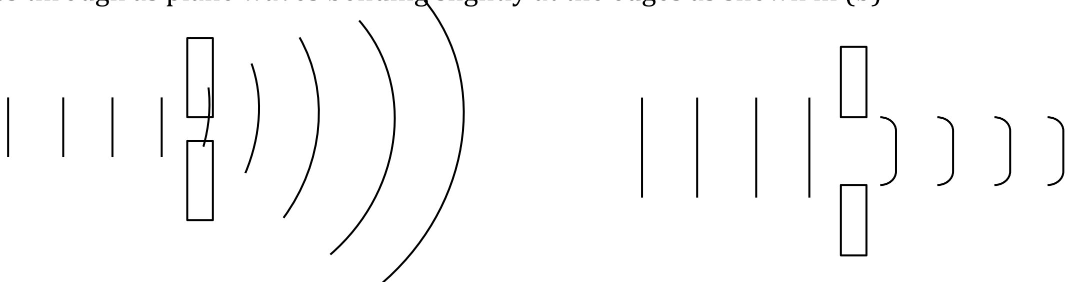
However, when the waves strike the boundary normally/perpendicularly, no bending occurs even though the speed and hence the wavelength changes.



Refraction of sound waves can be used to explain the long range of sound at night compared to daytime. This has been explained in the 'topic refraction of light'. TV and radio signals from a distant station also undergo a series of refraction and total internal reflection in the ionosphere towards the earth's surface making their reception possible.

9.8.3: Diffraction of waves

Diffraction may be defined as the spreading of waves behind an obstacle. When the aperture is nearly the same size as the wavelength of the waves, the waves emerge as circular waves spreading out around the obstacle as shown in (a) below. However, when the size of the aperture is relatively wider than the wavelength of the waves, the waves pass through as plane waves bending slightly at the edges as shown in (b).



a) Diffraction through a small aperture **b) Diffraction through a wide aperture**

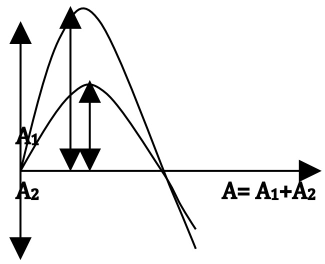
Diffraction of sound waves can be used to explain why sound within a room can be heard round a corner without necessarily having to see the source of the sound.

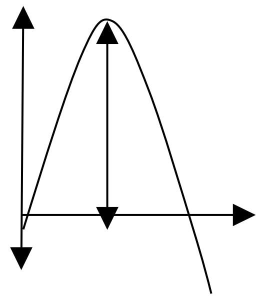
Diffraction of light waves is not a common occurrence due to their shorter wavelengths. Nevertheless, diffraction of light waves can be observed when light pass through a small opening at the roof of a dark room. A shadow which is broader than the opening forms on the floor of the room.

9.8.4: Interference of waves

Interference occurs when two waves merge. Such a merger may give rise to three cases:

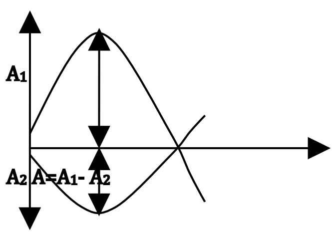
- A much larger wave is formed i.e. constructive interference.

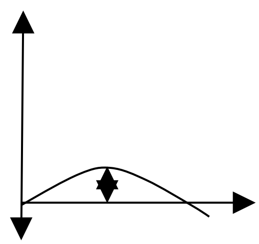




The waves are in phase and superimpose to produce a wave with a greateramplitude.

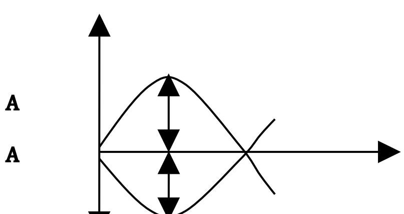
- A smaller wave is formed i.e. destructive interference.

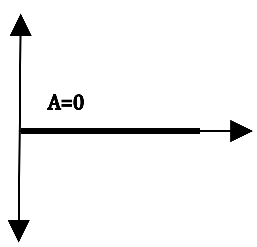




The waves are out of phase with a phase difference of 180^0. Since they have different amplitudes, they superimpose to form a wave with a smaller amplitude.

- A stationary wave.





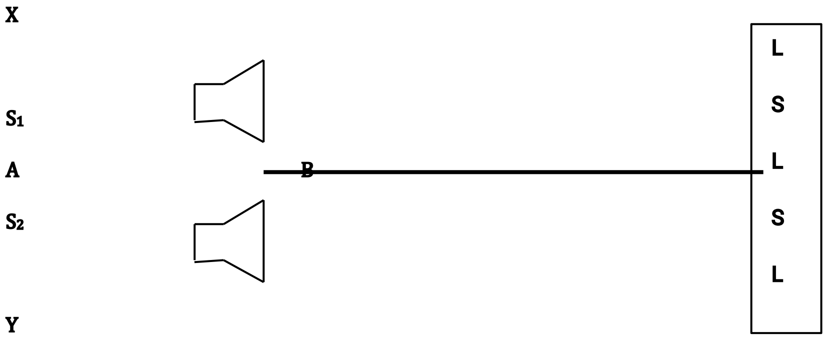
When the two waves which are out of phase with a phase difference of 180^0 superimpose, the result is a stationary wave having a zero amplitude.

Interference is a product of the **principle of superposition** which states: for two waves travelling in at a given point in the same medium, the resultant effect is the vector sum of their respective displacements.

- Interference of water waves can be shown by setting up two spherical dippers in a ripple tank which simultaneously generate waves. Alternating dark and bright radial lines will be observed on the screen representing regions of constructive and destructive interference respectively.

For interference to occur there ought to be a coherent source i.e. a source that generates waves of the same frequency and wavelength, equal or comparable amplitudes and having a constant phase difference.

- Interference of sound waves can be investigated by the set up below:

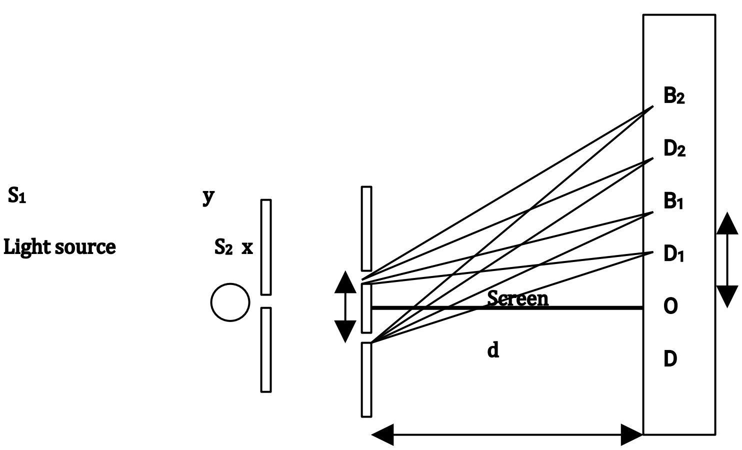


Two loudspeakers S_1 and S_2connected to an audio-frequency generator act as a coherent source. To an observer walking along a straight path XY, alternating loud and soft sound is heard. Along the line AB, a constant loud sound will be heard.

The regions with loud sound represent areas of constructive interference while the regions with soft sound represent areas of destructive interference. When the frequency of the signal is increased, the separation between the alternating loud and soft sound is reduced i.e. more close. Note that for a signal of any velocity, the higher the frequency the shorter the wavelength.

If instead the loudspeakers are connected such that the waves generated by one loudspeaker are exactly out of phase with those from the other, then all points along XY will have destructive interference and hence soft sound is heard throughout.

- Interference of light waves- this can be demonstrated by the Young's double slit experiment. Two narrow and very close slits S_1 and S_2 are placed infront of a monochromatic light source.



The light waves from the two slits undergo diffraction and superimpose as they spread out. A series of alternating bright and dark fringes are observed on the screen. The bright fringes are due to constructive interference while the dark fringes are due to destructive interference. However, along the central line through the centre of the slits and point O, it is bright throughout.

At O, the path difference of the two waves is zero since $S_1O=S_2O$. Moving upwards or downwards to the first bright fringe, the path difference is equivalent to one wavelength;

i.e**$S_2B_1-S_1B_1= 1\lambda$**

At D_1, the path difference is equivalent to half a wavelength;

$S_2D_1 - S_1D_1 = 1/2\lambda$

Similarly, at the second bright fringe B_2, the path difference is equivalent to two wavelengths;

i.e $\mathbf{S_2B_2 - S_1B_2 = 2\lambda}$

And $\mathbf{S_2D_2 - S_1D_2 = 3/2\lambda}$

Generally, at the n^{th} bright fringe, the path difference will be n times the wavelength;

$\mathbf{S_2B_n - S_1B_n = n\lambda}$

The wavelength of the light used can also be determined from the expression below:

$\mathbf{\lambda = xy/d}$,

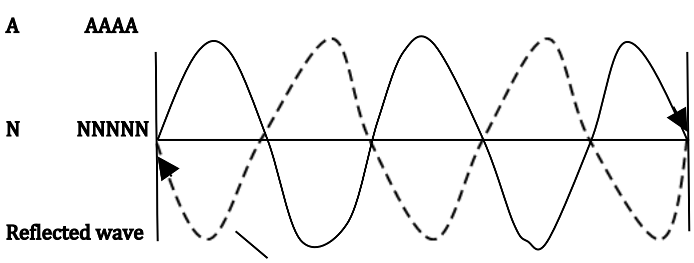
Where x- the slit separation,

y- Distance between successive bright fringes and

d- Perpendicular distance of the slits from the screen.

9.3: Stationary waves verses progressive waves

A progressive wave is a wave that continuously moves away from the source. When two progressive waves equal in amplitude and travelling in opposite directions superpose on each other, the resultant wave is referred to as **a stationary or standing wave**. It is a common occurrence in stringed instruments. When the string is plucked/played, a transverse wave travels along the string and is reflected back on reaching the other end of the string.



The points marked N are always at rest (zero displacement) and are called nodes while those marked A are where the wave has maximum amplitude (maximum displacement). They are called antinodes.

When two loudspeakers connected to the same audio-frequency generator are such that they face each other, then the two sound waves superpose to produce a stationary wave.

For two progressive waves to produce a stationary wave, the following conditions must be satisfied:

- They must be travelling in opposite directions.
- Must have same speed, frequency and same or nearly the same amplitudes.

The following table gives the comparison between a stationary and a progressive wave:

Stationary waves	**Progressive waves**
Do not move through the medium hence does not transfer any energy from the source.	Move through the medium transferring energy from the source to a point away.
The distance between successive nodes or antinodes is equal to 1/2λ.	The distance between successive crests or troughs is equal to the wavelength of the wave.

The amplitudes of particles between successive nodes are different.	The amplitudes of any two particles which are in phase are the same.

TOPIC 10.: GAS LAWS

Gas laws looks at the relationship between temperature, volume and pressure of gases.

10.1: Boyle's law

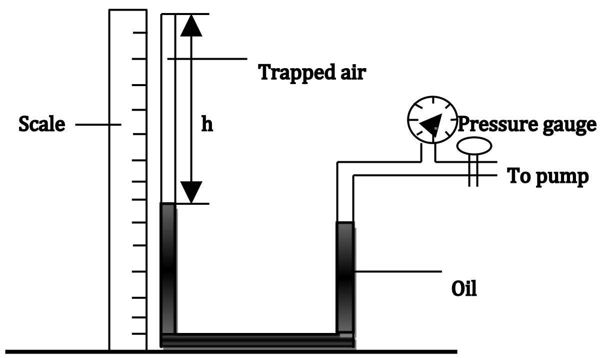
In this law, temperature of the gas is kept constant. Boyle's law states: the pressure of a fixed mass of a gas is inversely proportional to the volume, provided temperature is constant.

$P \alpha 1/V$

$P=k/V$

$PV=$ constant.

The following set up can be used to illustrate Boyle's law:



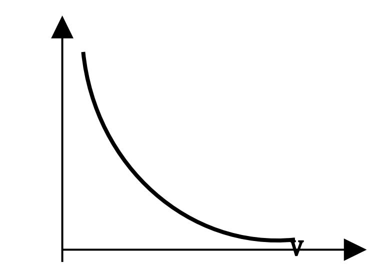
When pressure is exerted on the oil, the trapped gas (usually air) is compressed and the column h reduces. The pressure is measured using the pressure gauge. Since the cross-section area of the glass tube is uniform, the column h can be taken to represent the volume of the trapped gas (air).

Several values of pressure, P and volume, h are collected and recorded.

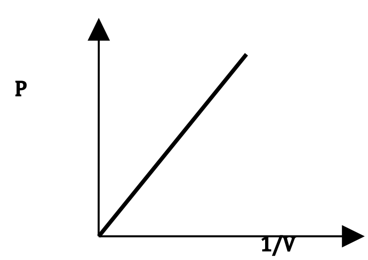
Pressure, P (Pa)	Volume, h(cm)	1/v (or 1/h m^{-1})	PV

A graph of pressure against volume is a curve as shown in (a) below:

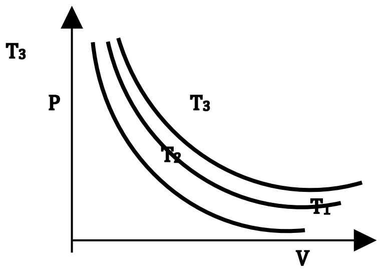
a)

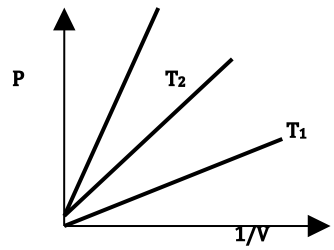


b)

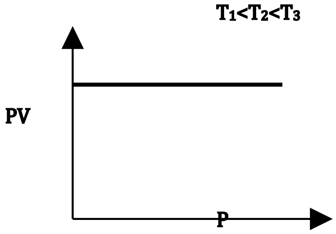


A graph of P against 1/V is a straight line through the origin as shown in (b) above while a graph of PV against P is a straight line parallel to the x-axis. If the experiment is repeated at different temperatures, similar curves to the above will be obtained. This isshownbelow:





$T_1 < T_2 < T_3$



Hence for a given mass of a gas, $P_1V_1 = P_2V_2$

Molecular explanation of Boyle's law

When a gas is put in a closed container, the gas molecules collide with walls of the container generating gas pressure. When the volume of the fixed mass of gas is reduced, the number of collisions per unit time and therefore the rate of change of momentum will increase. Consequently the gas pressure is raised. Hence a reduction in volume leads to an increase in the gas pressure.

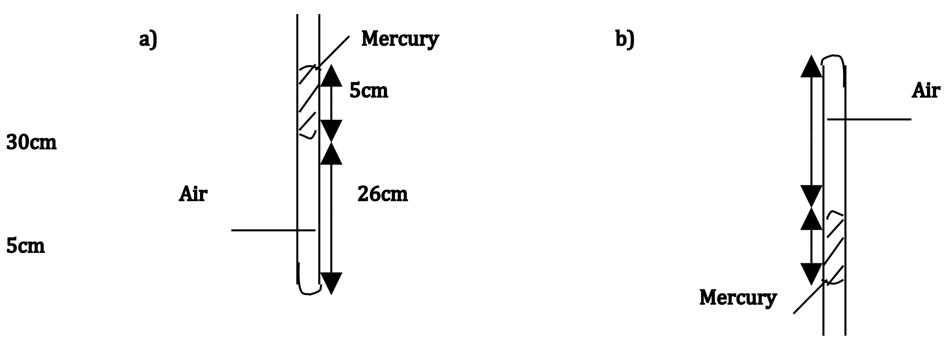
Example 10.1

1. A gas occupies a volume of $1.6cm^3$ at a pressure of $1.5 * 2^5$Pa. find the volume it will occupy at a pressure of $9.0 * 2^5$Pa if the temperature is kept constant.

   $P_1V_1=P_2V_2$

   $V_2= (1.5*2^5*1.6*2^{-6})/ (9.0*2^5) = 8.0*2^{-7}m^3$ or $0.8cm^3$

2. A column of air 26cm long is trapped by mercury thread 5cm long as shown in (a) below. When the tube is inverted as shown in (b), the air column becomes 30cm long. What is the value of the atmospheric pressure?



In (a), the gas pressure = $P_{Atm} + h\rho g$

In (b), the gas pressure = PAtm – hρg

Let the atmospheric pressure be x metres of mercury.

From Boyle's law, $P_1V_1=P_2V_2$

$(x+0.05)\rho g*0.26=(x-0.05)\rho g*0.3$

$0.26x+0.013=0.3x-0.015$

$0.04x=0.028$

$X=0.028/0.04$

$=0.7m$ (or 70cm)

Hence the atmospheric pressure=70cmHg.

3. The table below shows the results obtained in an experiment to study the variations of the volume of a fixed mass of a gas with pressure at constant temperature:

Pressure, P(cmHg)	60	……	90	……
Volume, cm^3	36	80	……	40

Fill in the missing values.

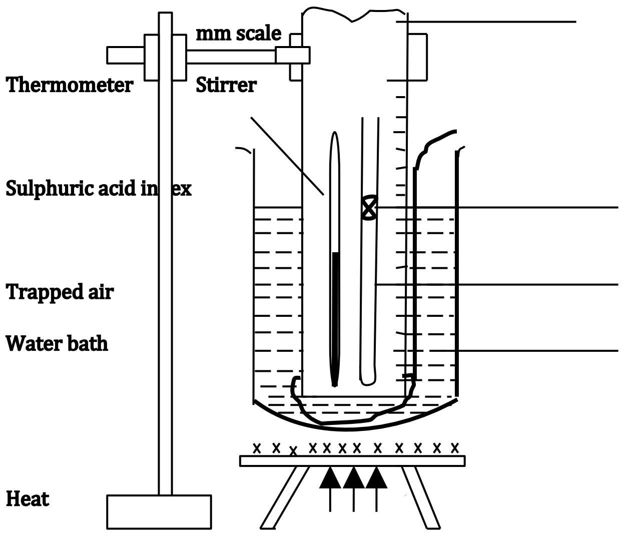
10.2: Charles' law

This law looks at the relationship between temperature and volume of a given mass of gas at constant pressure. It is obvious that when a gas is heated it expands i.e. increases in volume. The law states: the volume of a fixed mass of a gas is directly proportional to its absolute temperature provided the pressure is kept constant.

i.e. $V \alpha T$

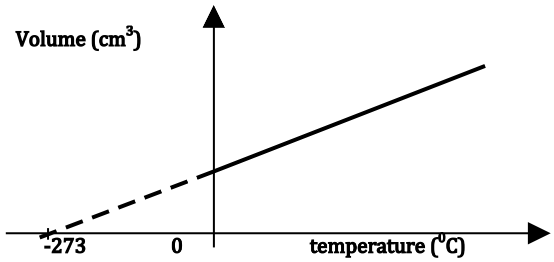
$V=kT$ or V/T = Constant

The set-up below can be used to verify Charles' law:

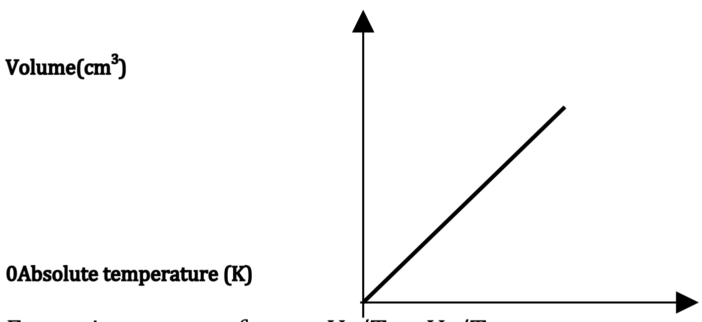


When the gas (trapped air) is heated in a water bath, it increases in volume. This is showed by an increase in the column h of the trapped air. Thus an increase in temperature of the gas causes an increase in its volume.

A graph of volume against absolute temperature appears as shown below:



If the graph is extrapolated, it cuts the x-axis at -273^{0}C. at this temperature, the gas is assumed to have a volume equals to zero. This is the lowest temperature a gas can ever fall to and is called the **absolute zero**. A temperature scale based on the absolute zero is referred to as the **absolute or Kelvin scale**. On this scale, the temperature must be expressed in Kelvin.



For a given mass of a gas, $V_1/T_1 = V_2/T_2$

This equation ONLY holds when the temperature is expressed in Kelvin.

Molecular explanation of Charles' law

When the temperature of a gas is increased, its molecules gain kinetic energy and move faster. This increases the rate of collision with walls of the container and hence increased pressure. However, since in Charles' law, pressure must be constant, the volume of the container must be increased accordingly so that the gas molecules can cover larger distance before colliding with the walls of the container. This would keep the gas pressure constant although its temperature is raised.

Example 10.2

1. A gas occupies a volume of $125cm^3$ at 15^0C and 755mmHg pressure. Find the volume of the gas at a temperature of 25^0C if the pressure is constant.

   $V_1/T_1 = V_2/T_2$

   $125/(15+273) = V_2/(25+273)$

   $V_2=(125*298)/288 =129.34cm^{9.}$

2. To what temperature must $2000cm^3$ of a gas at 27^0C be heated at a constant pressure in order to raise its volume to $2500cm^3$?

   $V_1/T_1 = V_2/T_2$

   $T_2=(2500*300)/2000 =375K$ or 22^0C.

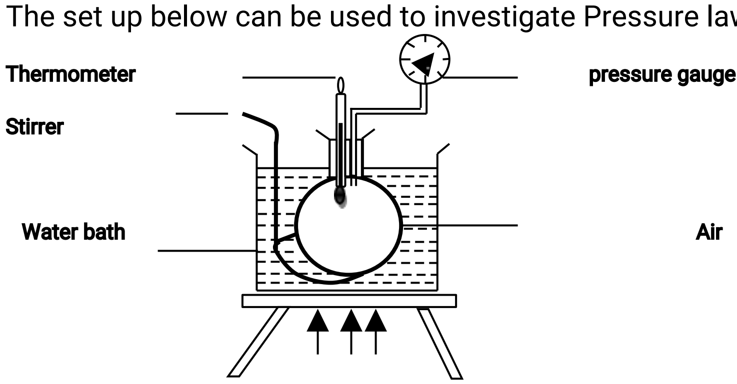
10.3: Pressure law

Raising the temperature of a fixed mass of a gas at a constant volume increases the average kinetic energy of the gas molecules. Pressure law states: ***the pressure of a fixed mass of a gas is directly proportional to its absolute temperature at a constant volume***;

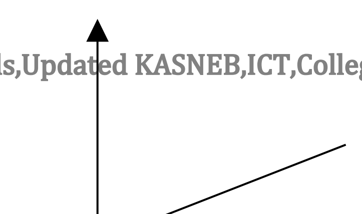
$P\alpha TP=kT$ or $P/T=k$

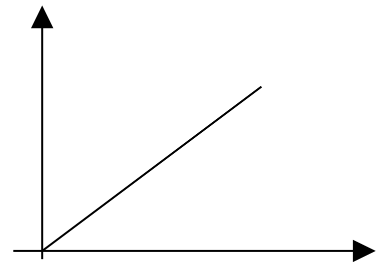
Thus at constant volume, $P_1/T_1= P_2/T_2$

The set up below can be used to investigate Pressure law:



Several values of temperature and the corresponding pressures can be collected and used to plot a graph of pressure against absolute temperature. The graph will appear as shown below:





-273 0 Temperature (^0C) 0 Absolute temperature (K)

Example 10.3

1. A tin closed with an airtight lid contains air at a pressure of $1.0*2^5$Pa at a temperature of 12^0C. If the temperature at which the lid opens is 88^0C, determine the pressure attained by the gas.

   $P_1/T_1= P_2/T_2$

   $P_2=[1.0*2^5*361]/285$ =126,668.67Pa

The three laws combined can be expressed as;PV/T =constant, kOr simply

$P_1V_1/T_1= P_2V_2/T_2$

The above equation is referred to as the **equation of state**. In general for a fixed mass of a gas, PV/T=a constant. If 1 mole of the gas is used, then;

PV/T= R, where R is the **universal gas constant**.

Example 10.4

1. A gas occupies a volume of $200cm^3$ at 25^0C and 760mmHg. Find its new volume at -23^0C and 750mmHg.

   $P_1V_1/T_1= P_2V_2/T_2$

$V_2=[P_1V_1T_2]/P_2T_1$=[760*200*250]/[750*298] $=170cm^3$